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THEORETICAL ESSAYS ON SUSTAINABILITY AND ENVIRONMENTAL POLICY

JOHN PEZZEY

PHD THESIS

THEORETICAL ESSAYS ON SUSTAINABILITY AND ENVIRONMENTAL POLICY

John Charles Vincent Pezzey

A thesis submitted to the University of Bristol
in accordance with the requirements of the degree
of Doctor of Philosophy in the Department of Economics
in the Faculty of Social Science

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Abstract

This thesis is in two parts. In Part I, sustainable development (SD) is defined as any development path where the instantaneous utility of a representative person is forever non-declining. This definition is applied to three models of economic growth based on privately-owned, non-renewable or expanding exhaustible resources, after an extensive debate of the appropriateness of such modelling frameworks. In the first, continuous-time model, the total resource stock has amenity value. The environmental policy which internalises this amenity value is shown to improve sustainedness, but to remain distinct from SD policy. The second model, also in continuous time, has no externalities. It shows that non-declining aggregate wealth does not guarantee SD, because wealth is measured at 'unsustainable' prices; and that if the non-declining utility constraint bites, the SD path which maximises present value differs at all times from the conventionally 'optimal' path. Both models show that SD policies will be hard to carry out in the long run, since they must eventually use large subsidies in order to encourage more saving.

The third model uses a discrete, non-overlapping generational structure to study intergenerational transfers of an expanding, exhaustible resource. It studies the effect of 'mating-bequest externalities' when children marry at random. It shows that such externalities can justify a collective SD policy, since parents may desire SD, but be unable to achieve it efficiently, or even at all, by increasing their individual bequests to their children.

Part II explores a different question: How can market mechanisms of environmental policy be used to reduce the overall cost of pollution control in a heterogeneous industry in a politically realistic way? We first show that pollution charges and/or subsidies are fully symmetrical to marketable pollution permits, provided that symmetrical assumptions are made about the property rights built into both mechanisms. We then use this symmetry to show how the overall cost savings from market mechanisms can be divided among environmental users, taxpayers and polluters so as to give them each a strictly positive benefit, without losing either short or long run efficiency.

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My first and foremost thanks goes to my two supervisors, David Ulph till January 1992 and Geoff Lewis since then, for all their helpful advice and hard work on my behalf. I am particularly grateful to Geoff Lewis for his careful reading of the final version, although any errors that remain are of course entirely my responsibility.

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For providing enduring and invaluable moral support over the years, my heartfelt thanks go to my parents Shelagh and Frank, and to Kerry Chester, David Cope, Andy Fagg, Chris and Helena Finden-Browne, David Fleming, Carol Freeman, Caroline Gordon, Pete Lund, Adele Morris, John Proops, Rob Spurr, Tricia Stokowski and Mark Thompson.

I include thanks for substantive comments that are specific to particular chapters as leading footnotes in them. In addition, I especially wish to thank Chuck Howe and Ray Prince for providing the original encouragement to embark on this venture, and to Rich Howarth and Mike Toman for many long, detailed conversations which have been outstandingly useful.

I hope I can find ways of repaying you all for your help and encouragement.

Declaration

I, John Charles Vincent Pezzey, declare that all of this thesis is my own original work. None of the work in this thesis has been submitted for any other degree. Any views expressed here are mine, not those of the University of Bristol. This thesis is approximately 72,000 words long, including all preliminary pages, lists of references, etcetera.

Signed.. John C.V. Pezzey

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PREFACE

This thesis deals with two distinct topics: the theory of sustainable development, and the political economy of market mechanisms of pollution control. Both topics are clearly part of environmental and resource economics, and they have some broad similarities. Both overlap with ethics and politics, because they involve questions of distribution: intergenerational distribution in the case of sustainable development, and intragenerational distribution in the case of market mechanisms. My treatment of both topics will, among other things, study the effect of taxes aimed at internalising the externalities caused by environmental resource stocks and flows, in order to achieve some social objective. And my study of market mechanisms will reveal important political limitations on the frequently recommended policy of using environmental taxes to make development sustainable, because of the effect that such taxes would have on intragenerational equity.

However, at the detailed level of analysis, there is little similarity between my investigations of sustainable development theory and of market mechanisms. The former is dynamic, and assumes that both consumers and firms are uniform. The latter is comparative static, and assumes that both are heterogenous. There is very little overlap between the literatures on the two subjects. The thesis is therefore organised into two parts, each with its own introductory and concluding chapters: **Part I** on sustainable development, and **Part II** on market mechanisms. Such common ground as exists between the two Parts will be noted where appropriate.

PART I

CHAPTER 1

INTRODUCTION TO PART I

1.1 THE SUSTAINABLE DEVELOPMENT (SD) AGENDA

For nearly two hundred years, there have been prophets of doom caused by resource scarcity. Malthus (1798/1976) predicted that human population growth would outstrip food supply. Jevons (1865/1977) concluded that the rapid increase in coal consumption, coupled with the finite nature of the supply of coal, would cause progress to stop in the near future. The computer simulations of Meadows *et al.* (1972) predicted that diminishing natural resources and increasing pollution might well cause spectacular global declines in both population and living standards by the end of the next century. In their various ways, all these writers effectively claimed that the rapid expansion of human civilization occurring at the time they were writing would be *unsustainable*, because of the environmental resource limits imposed by a finite planet. That is, they were not just concerned that there would be 'limits to growth'. They also feared that once limits are hit, growth will go into reverse. And in various ways they all felt that following an unsustainable growth path would be unfair to future generations; or in the economist's jargon, would be a violation of intergenerational equity.

Yet, for various reasons, it was not until the publication in 1987 of *Our Common Future* by the World Commission on Environment and Development (WCED), that sustainability and especially *sustainable development* (**SD**) became the watchwords of concern about the adequacy of finite environmental resources to provide for the long term future of civilisation. **Table 1.1** reveals the extent to which these words came into

Table 1.1 Changes in number of published journal articles and book reviews indexed in SSCI, 1984-1993 (1984=100)

	Year	'84	'85	'86	'87	'88	'89	'90	'91	'92	'93
Word in title	sustain...	100	98	127	123	123	215	252	344	438	550
	industr...	100	101	103	102	96	92	92	85	86	95
	financ...	100	96	103	100	94	94	102	92	93	108

academic fashion. It lists the annual numbers of journal articles and book reviews containing words starting with "sustain...", with "industr..." and with "financ..." in their titles, that were logged by the Social Science Citation Index (SSCI) during 1984-1993. Note the quadrupling of publications on sustainability after 1987, as compared to little or no growth of publications on the other two topics.

As well as simply reviving concern about the importance of environmental resources for the long term future, WCED (1987) (and the major public consultation exercise that preceded it) also shifted it in a major new direction. The 'limits to growth' debate of the early 1970s had mainly focused on the harmful effects of *excessive* per capita economic development, and its associated depletion of *non-renewable* resources, on the future of *rich* countries. Typical features of this problem were held to be the sheer exhaustion of mineral and energy resources, and the environmental pollution damage caused by the material flows of such resources. In contrast, WCED's main focus was on the harmful effects of *inadequate* per capita economic development (i.e., poverty), and its associated depletion of *renewable* resources, on the long term future of *poor* countries. WCED's analysis suggested that poverty typically causes renewable resources in the natural environment to be (a) very important to the economy; (b) protected only by common property management systems, which were breaking down into open access; (c) managed using shortened time horizons; and (d) difficult to improve and protect because of the general shortage of investment funds.[1] WCED therefore asserted that paying attention to the *distribution* of income – both within countries, and between countries – is a vital part of making development sustainable. This assertion is partly tautological, in that a more equitable income distribution is seen by many as one of the goals of development itself; but it also contains a testable hypothesis, that inequitable development is likely to be unsustainable.

However, WCED's analysis is often far from rigorous. Reading it frequently raises fundamental questions about the economic theory of SD. In this thesis I will study seven of these:

Q1: What is the meaning of SD, particularly in comparison to the conventional economic concept of socially PV-optimal[2] growth?

Q2: When are socially PV-optimal or free market development paths also SD paths?

Q3: When they are not, what policies can achieve SD, and how do these policies relate to environmental policies?

Q4: In particular, what is the socially PV-optimal SD path, and what policies can achieve it?

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1. A neat summary of this view is given by the World Bank (1992, p2): "Without adequate environmental protection, development will be undermined; without development, resources will be inadequate for needed investments, and environmental protection will fail."

2. Throughout the thesis the word *PV-optimal* rather than just 'optimal' will be used to describe the conventional objective of development or growth policy of maximising discounted present value (**PV**) over time. As shown in Chapter 2, this extra jargon is essential, since the thesis explores other goals of intertemporal social policy (such as PV-optimal SD in Q5) which one might wish to adopt.

Q5: Can a policy goal of achieving SD be justified in terms of personal preferences?

Q6: Is there a simple indicator, based on currently available information, of when an economy is sustainable or unsustainable?

Q7: How are the answers to Q2-Q6 affected if an economy's standard of living (i.e. consumption per capita, in my models) is historically given at the starting time? (I refer to any such effects as *initial condition* effects.)

However, I will *not* analyse these questions in the context of WCED's main agenda of renewable resources, poverty, and the alleged need to grow in order to protect the environment. I will consider non-renewable resources, and what I will at times call 'expanding' resources (to be explained below), but not genuinely renewable resources. I will ignore all questions of intragenerational distribution and equity, any effect of poverty on discount rates, and any possibility of investing to restore the environment, other than refraining from degrading it in the first place. I will not distinguish 'growth' from 'development', a distinction which is important in many contexts, as Daly (1990, p1) rightly points out. My agenda is therefore more that of the early 1970s than the late 1980s and early 1990s.

The historical reason for these choices is simply that they arose naturally out of studying the 1970s literature. But they are also choices which can be defended, as follows. Questions Q1-Q7 are still relevant, although perhaps not topics of pressing popular concern, to non-renewable resource depletion in rich countries, as well as to renewable resource depletion in poor countries. Non-renewable resources are probably still more important than renewable resources in rich countries, at least for medium-term sustainability, if not ultimate survival. The aftermath of the limits to growth debate in no way provided all the answers about sustainability and non-renewable resources. One reflection of this is that of recent articles on sustainability (again as indexed in SSCI) covering specific sectors, a good proportion cover sustainable energy, transport, cities and industry (even though these concepts may be loosely defined). Also, the literature on PV-optimal growth with non-renewable resources provides a striking example of unsustainable development (Dasgupta and Heal 1974, to be discussed at length in Chapter 3).

Another reason for my choice is undeniably tractability. It is harder to model SD policy rigorously for an economy reliant on a renewable resource, other than in the simple case where the resource has no substitute. I explain this in more detail below. First, though, Section 1.2 tackles question Q1, and explains why SD is broadly defined here as non-declining well-being. It also notes that technical progress or capital-resource substitutability[3] are assumed, which means that non-declining well-being does not require non-declining natural resource stocks. Section 1.3 explains why I define well-being more precisely as the instantaneous utility of a representative person, and also notes the assumptions of an infinite time horizon and deterministic behaviour. Section 1.4 then discusses the avoidance of renewable resources in detail. Finally, Section 1.5 gives an overview of Chapters 2, 3 and 4, which contain the substantive research results of Part I.

3. I use 'capital' solely to refer to human-made, not natural assets; see footnote 2 of Chapter 3 for more detail.

1.2 DEFINING SD, I: NON-DECLINING WELL-BEING

Sustainability and SD are concepts which are notoriously hard to define in a precise way that commands broad agreement. My review six years ago found dozens of different definitions (Pezzey 1989). Now, thanks to the explosion in sustainability writing shown in Table 1.1, there would doubtless be hundreds. But doing an exhaustive up-date of this review would serve little purpose, as few new issues have arisen since then. Instead I will explain the definition of SD used in this thesis, and compare it to some of its main rivals in three ways (in Sections 1.2.1, 1.2.2 and 1.2.3 respectively). Firstly, my definition is a single criterion, rather than many loosely linked criteria. Secondly, it does not require non-declining natural resource stocks. Thirdly, it is not the same as non-declining aggregate wealth. The first two comparisons can be rapidly explained here, but the third is not at all obvious and will be a major topic of inquiry in Chapter 3.

1.2.1 A single criterion for SD is used, rather than several

One of the main reasons why sustainability definitions are both prolific and frequently vague is, I contend, because many writers refuse to accept the existence of trade-offs between several desirable criteria which they would like to see SD achieve. In particular, both environmental protection and a more equal (int*ra*generational) distribution of income are popular policy aims in their own right, and also aims whose achievement *may* improve int*er*generational equity. This leads many writers to use sustainability or SD merely to *relabel* an existing, conventional policy goal. As illustrated at the start of Chapter 2, many 'environmentally correct' or 'environmentally sound' policies for transport, energy, forestry, etc, have become relabelled *en masse* as policies for 'sustainable' transport, energy or forestry, without any real change in their content, apart from a greater awareness that the problems to be addressed will probably get worse over time.

Somewhat distinct from relabelling is the way that many writers rather vaguely define SD as 'containing', 'comprising', 'entailing' or 'being consistent with' intergenerational equity *and* environmental protection *and* intragenerational equity. This multicriteria approach was greatly boosted by the best known passage in WCED (1987, p43):

> "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. It contains within it two key concepts:
> - the concept of 'needs', in particular the essential needs of the world's poor, to which overriding priority should be given; and
> - the idea of limitations imposed by the state of technology and social organisation on the environment's ability to meet present and future needs."

But is a particular development path sustainable if, by achieving greater equality among nations in 2000, it decreases the availability of fossil fuels and minerals in 2100, and increases global warming and the destruction of the ozone layer then? Is development sustainable if it entails a massive expansion of renewable energy systems which makes life better in 2100, because of the fossil fuels that will be saved, but is bad for the environment because of all the windmills and dams that have to be built? A multicriteria approach cannot readily answer such questions, because it says little about how to make unavoidable *tradeoffs* among the two types of equity and environmental quality.

To avoid both relabelling and multiple criteria, I focus on the intergenerational equity of SD, and define it solely as *non-declining (per capita) well-being forever.*[4] An environmental policy will then be an SD

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4. This is a mainly 'welfarist' approach. A 'resourcist' approach would define SD as a forever non-declining *opportunity* for well-being, irrespective of what future

policy only insofar as it helps to achieve non-declining well-being (for example, because well-being depends among other things on environmental quality). The same would apply to a redistributive policy within the current generation, although for simplicity's sake, I *ignore all questions of intragenerational equity* throughout the rest of the thesis.

The non-declining well-being approach is in line with a recent quotation from David Pearce (although as noted below and by Beckerman 1994, pp 194-5 notes, earlier quotations by Pearce take a rather different line):

> "'Sustainability' therefore implies something about maintaining the level of human well-being so that it might improve but at least never declines (or, not more than temporarily, anyway). Interpreted this way, sustainable development becomes equivalent to some requirement that well-being does not decline through time." (Pearce 1993, p48)

There is also an important technical distinction to make here between sustainability and SD. An economy is sustain*able at a point in time* if, given some level of well-being and endowments at a point in time, it *can* achieve non-declining well-being thereafter until the end of time. It is sustain*ed* or following a path of *sustainable development over time* if it *does* achieve non-declining well-being for all time. It is harmless to overlook this distinction in much general writing on the topic, but it is important for some aspects of the analysis in Chapters 2 and 3. For example, an economy may currently be sustainable, but embarked an unsustainable development path.

'Sustainedness' and SD thus both mean non-declining well-being for all time, and all three terms will be used interchangeably below. **Figure 1.1**, where the economy follows a falling, then rising, then falling path of well-

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generations actually do with that opportunity at any instant (Dasgupta 1990; Broome 1992, Chapter 2). But note that a rigid constraint that well-being must not decline still falls short of a purely welfarist approach to SD.

being over time, shows why this makes sense. Suppose U_1, well-being at time t_1, is below U_1^m, the maximum constant level of well-being which the economy can sustain from t_1 until the end of time, as shown. Then the economy is sustainable at t_1, but not developing, since well-being is falling. And if U_2, well-being at time t_2, is above the maximum sustainable level U_2^m then, as shown, then the economy is developing but not sustainable at t_2, i.e. well-being must eventually decline. If the whole path is feasible and never has declining well-being, i.e. is sustained, then it is an SD path.

I will make the meaning of 'well-being' more precise in Section 1.3 (so far, the only thing clear is that it should measure some psychological state at a point in time[5]), and I will discuss its moral and technical aspects in Chapter 2. First, though, I need to point out that because of assumptions I will make about substitutability, my definition of SD does not require non-declining environmental resource stocks. It is also, for other, less obvious reasons, not equivalent to non-declining aggregate wealth.

1.2.2 SD is assumed to be possible even with declining environmental resource stocks

Many writers on SD assume that, below certain threshold levels, the roles that environmental resources perform in the economy are in practice *non-substitutable*.[6] This in turn means that SD, defined as non-declining well-

5. This is why I avoid the word 'welfare' here, and use it only to mean some aggregation of well-being *over* time. This avoids the serious confusion which Beckerman (1994) causes when he uses welfare to mean both well-being at a point in time, and its aggregation over time.

6. The qualifier 'in practice' in the previous sentence recognises that one might have an economy with labour and resources being substitutable for one another in the production function, but if the labour supply is fixed, then there is no *available* substitute for declining natural resource inputs. SD would then require non-declining

being, amounts to a requirement that environmental resource stocks be preserved. For example:

> "...there are strong reasons to think of sustainable development as involving a further constraint, namely that the stock of environmental assets should not decrease." (Pearce, Markandya and Barbier 1989, p48)

> "For the management of renewable resources there are two obvious principles of sustainable development. First that harvest rates should equal regeneration rates (sustained yield). Second that waste emission rates should equal the natural assimilative capacities of the ecosystems into which the wastes are emitted. ... ...it is possible to exploit nonrenewable [resources] in a quasi-sustainable manner by limiting their rate of depletion to the rate of creation of renewable substitutes." (Daly 1990, pp2,4)

Another related view is that in SD "the objective is to reach minimum divergence between acceptable and realised levels of relevant indicators [of sustainability]" (van den Bergh, 1993, p400), where the acceptable levels of relevant indicators would probably be related to thresholds of non-substitutability. By contrast, I assume throughout the thesis that *SD is feasible even with declining resource stocks and/or flows*, thanks to technical progress and/or increases in capital services which can substitute for declining resources. I must stress that this assumption is for the sake of analytical clarity, *not* because I believe that endless substitution possibilities do in fact exist for all resources. The substitutability of human-made for environmental resources in production and consumption is an important but highly complex *empirical* question (summarised in Pezzey 1992, pp 337-340). Ideally I would have wished to build limits to substitutability into some of my models, as noted in Section 3.3.2 of Section 3. But I do not

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resource stocks, as in the first model in Mourmouras (1993). Another semantic point is that one cannot describe non-substitutable natural resources as 'essential', since this word is reserved for other meanings – although there is in fact a conflict between the definitions of essentiality given in Dasgupta and Heal (1974, p25) and in Dasgupta and Heal (1979, p198).

subscribe to Daly's view that the substitutability question can be resolved by purely logical reasoning, as illustrated by:

> "So capital can substitute for resources in the limited domain of minimizing and recycling waste pieces of materials in process. But this substitutability is *trivial* compared to the *overwhelming* complementarity that must *necessarily* exist between that being transformed (resource) and the agent of transformation (capital)." (Daly 1990, p3; emphasis added)

How can one know that substitutability is trivial, and overwhelmed by complementarity, without some attempt to measure it?

1.2.3 Non-declining aggregate wealth does not guarantee sustainability

In other places in the same book as cited above, Pearce et al are willing to assume that capital is substitutable for resources. They then usually also assert that *non-declining aggregate wealth* (the combined value of the stock of man-made assets and environmental assets) is a condition for achieving sustainability. For example:

> "...we can meet our obligations to be fair to the next generation by leaving them an inheritance of wealth no less than we inherited. Moreover, so long as each single generation does this, no single generation has to worry about generations far into the future." (Pearce, Markandya and Barbier 1989, p35)

This clearly describes non-declining wealth as a *sufficient* condition for non-declining well-being, but elsewhere Pearce describes it as a *necessary* condition: "Sustainable growth and development cannot be achieved if [wealth] is declining." (Pearce 1993, p49). Elsewhere still, he is unclear about sufficiency or necessity, but generally one gets the impression that he regards the condition as both necessary and sufficient, so that non-declining well-being and non-declining wealth are equivalent.

In Pezzey (1989, p19 and 1992, p342) I took this equivalence for granted, since not only Pearce but also Solow (1986) and Maler (1991) had made this assertion. But my analysis in Chapter 3 will show that the sufficiency condition is *not* in fact true. One can have non-declining wealth and yet a level of well-being which is unsustainable, i.e. bound to decline in the future. Both theoretically, and for practical applications such as resource accounting and sustainability indicators, this is probably the most important result of the thesis. More details will therefore be given soon, in the preview of Chapter 3 in Section 1.5. First, however, let us define more precisely *whose* well-being *when* should be non-declining on an SD path.

1.3 DEFINING SD, II: NON-DECLINING UTILITY OF A REPRESENTATIVE PERSON

Suppose that the world comprises overlapping cohorts or generations, each comprising identical people, so that for simplicity, we can represent each generation by a single person who lives $N+1$ periods. We thus ignore population growth, inequity amongst people of the same age, and any differences in lifespan. Adapting the notation developed by Burton (1993), suppose that

$\boldsymbol{c}_t^a$	is the vector of consumption goods received by a person of age a in period t (a person has age 0 in the first period of her life)
$\boldsymbol{S}_t$	is the vector of total environmental resource stocks in period t
$\mu_t^a(\boldsymbol{c}_t^a,\boldsymbol{S}_t)$	is the instantaneous utility perceived by a person of age a in period t[7]

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7. I will shortly equate 'well-being' with measures solely based on a person's subjectively-perceived utility μ_t. In so doing I sidestep the important debate (in, for example, Anand and Ravallion 1993) about whether a more objective index of human development such as health or literacy is a better measure of 'well-being' than utility.

$U_t(\mu_t^0, \mu_t^1, \ldots, \mu_t^N)$ is the net benefit to society in period t

$V_t(\mu_t^0, \mu_{t+1}^1, \ldots, \mu_{t+N}^N)$ is the "lifetime welfare" of a person born in period t

(Unfortunately, I have had to interchange the meaning of Burton's U and V, so that I can use U in Chapters 2 and 3 for the instantaneous utility of a representative person, and thus be consistent with the standard notation in PV-optimal growth models.)

Such a formulation is already highly restrictive because it excludes any recognition that people's well-being is strongly influenced by *relative* variables, such as their consumption relative to what it was in earlier periods, or what their peers' consumption levels are now (see Pezzey 1992, pp 351-4 for a discussion of these influences). But even with such restrictions, there is an immediate question of whether the net benefit to society (U), or a new-born person's lifetime welfare (V), should be the 'well-being' that is non-declining over time on an SD path. Since SD is at heart a concept of inter*generational* equity, the latter measure is clearly more appropriate. Preventing falls in net benefit U may conceivably amount to preventing falls in people's instantaneous utilities at some stage of their lives. These falls might be part of those people's plans to maximise their own lifetime welfare V, and there is no obvious reason for considering such falls unfair to the next generation. But for V to fall from one value of t (which defines a generation) to the next clearly does raise a problem of intergenerational equity.

I should therefore ideally like to have built an overlapping generations model, with SD defined as non-declining lifetime welfare V_t. The obvious function to use for V_t would be the present value (PV) formulation

$$V_t \quad = \quad \sum_{a=0}^{N} \mu_t^a/(1+\delta_i^t)^a \qquad \text{where } \delta_i^t>0 \text{ is the } \textit{personal lifetime} \text{ utility discount factor}$$

Such a modelling framework would allow us to distinguish clearly between intertemporal self-interest, which may cause people to conserve resources over time so as to have something to trade for others' labour power when they are retired; and intergenerational altruism, which may cause people to conserve resources as if they were maximising an intergenerational social welfare function (ISWF) like

$$W_t \quad = \quad \sum_{x=0}^{\infty} V_{t+x}/(1+\delta_g)^x \qquad \text{where } \delta_g>0 \text{ is an } \textit{intergenerational} \text{ utility discount factor (assumed constant).}$$

This framework also reveals another problem of 'resourcism' versus 'welfarism': if lifetime welfare V_t falls from generation to generation simply because the discount factor δ_i^t increases over time, can this be regarded as a problem of intergenerational equity? (Arguably not, but in effect I duck the problem by assuming a constant $\delta_i^t=\delta_i$ for all t.)

However, in this thesis, I will *ignore the age structure of the population alive at any time.* I will instead represent the population by a single person, whose well-being is measured by an instantaneous utility function $U(t)=U(c_t,S_t)$, which has a constant functional form over time. *Sustainedness* is then defined as *non-declining (instantaneous, per capita) utility U(t)*, henceforth **NDU**, and *sustainable development* is *NDU for all time.* In Chapters 2 and 3, time is continuous and the time horizon is infinite. One may then consider society as represented either by an infinite series of infinitesimally-lived people, or as a single, immortal, infinitely-lived person. As Burton points out, this is equivalent to assuming a common factor $\delta_i=\delta_g=\delta$ for both personal lifetime and intergenerational discounting. In Chapter 4, time is discrete and comprises an infinite series of non-overlapping generations, each represented by a single couple who live for one period. Personal lifetime discounting then disappears altogether. In either the continuous or discrete time case, my representative person framework leaves out much realism and richness in economic and policy mechanisms, such as retirement, bequests, and all trades and transfers between contemporaneous young and old people.

My primary reason for making this choice is, as with the decision to avoid renewable resources, to improve tractability. Tractability was also why I chose not to adopt the criterion in Riley (1980), which is effectively to define sustainedness as non-declining present value in an immortal, continuous time framework, i.e. as non-declining $\int_t^{\infty} U[c(x),S(x)]e^{-\delta x}dx$ over time t. At face value this criterion has neither much analytic simplicity, nor a clear identification of separate generations. A secondary reason why I define sustainedness as NDU is to produce results comparable with the well-known literature on exhaustible resource allocation over time. Almost always, this literature also uses the representative person framework to avoid the difficulty of working with overlapping generations. Such literature (mostly using continuous time and an infinite horizon) comprises not just the PV-optimal growth literature, such as the classic papers by Dasgupta and Heal (1974) and Stiglitz (1974) and the vast majority of PV-optimal growth models in journals such as the *Journal of Environmental Economics and Management*, but also most of the intergenerational 'equity'[8] literature stemming from seminal papers by Solow (1974) and Hartwick (1977), and some recent technical literature on sustainability such as Klaassen and Opschoor (1991).

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8. This is in quotation marks because 'equity' normally suggests a wide range of notions of justice and fairness, but Solow and Hartwick restrict it purely to mean *equality* of consumption, and hence utility, over time.

Losing all age structure is nevertheless a major and rather disappointing methodological choice. It is therefore worth further illustrating the problem of tractability, by summarising what Howarth and Norgaard (1990, 1992, 1993), Howarth (1991a, 1991b, 1992) and Mourmouras (1991, 1993) have and have not achieved with an overlapping generations approach to economic growth based on exhaustible resources. The main features of their papers are summarised in **Table 1.2**. This reveals that none of Howarth's papers considers SD policies as such. This appears to be because the complexity of the finite overlapping generations structure means that such policies are impossible to calculate, except by numerical solution of a 2-generation example with a logarithmic functional form for utility and a Cobb-Douglas production function. The infinite number of generations assumed by Mourmouras allows him to get somewhat further, and calculate analytic policies which do achieve constant lifetime utility across generations. But to do this he has to assume the same specific functional forms. He is also crucially reliant on assuming an exponentially growing rather than a strictly non-renewable resource, and (in the cases where he considers capital investment) a capital stock which decays completely in one generation.

By using the greater tractability of the representative person framework, I will be able to show some more general properties of SD policies for exhaustible resources. For example, such policies can be different from environmental policies, they may not be needed all the time, and they may be difficult to carry out in the very long term. To best guide policy in the real world, results from both overlapping generation and representative person models will have to be used; neither of them has any clear overall superiority.

Table 1.2 Main features and conclusions of intergenerational models of growth and exhaustible resource depletion

Article	How & Nor 90	How 91a	How 91b	How & Nor 92	How 92	How & Nor 93	Mour 91	Mour 93
Type of exhaustible resource	Non-renewable	Non-renewable	Non-renewable	Degradable, self-repairing	Exponentially growing	Non-renewable	Exponentially growing	Exponentially growing
Number of generations	2	G, 2	G, 2	T	∞	2	∞	∞
Number of periods in generation	2	2	2	2	1	2	N	N
Utility discounting within a generation	✓	General; –	–	–	–	–	✓	✓
Intergenerational altruism	–	–	–	–	–	✓	–	–
Intergenerational social welfare functions (ISWF)	Utilitarian; maximin	(Qualitative only)	Maximin	Discounted utilitarian	–	Laissez faire; maximin; utilitarian	–	Maximin
Role of labour	Direct service	Factor input	Factor input	Factor input	–	Factor input	Factor input	Factor input
Reproducible capital	–	✓	✓	✓	–	✓	Sometimes; if so, decays in 1 period	
Additional features	–	–	Uncertain technical progress	Externality from cumulative pollution	Subsistence consumption level	Externality from future social utility	–	–
Intergenerational rights/transfers of:	Resources	Produced good	Produced good	Produced good	Resource	Produced good	–	–; produced good; resources
Other policy instruments	–	–	–	Pollution tax	–	–	–	Resource flow tax + stock subsidy; resource purchase + income tax
Main conclusions	(1): Intergenerational property rights/transfers affect each generation's efficient (i.e. maximum) utility level, and hence affect intertemporal social welfare. Socially optimal rights/transfers therefore depend on choice of ISWF; in general, it's suboptimal for current generation to have all resource rights.		As (1), and: If future gens. in difft. states of nature are distinct, socially opt'l transfers may be risk-averse and not unconditionally efficient.	As (1), and: Efficient pollution tax and sustainability rise, and interest rate falls, if utility discount rate lower; so envt. valuation depends on the ISWF chosen.	Justice among contemporaries + chain of obligation + subsistence requirement $\Rightarrow$ consump. must be constant over time.	As (1), and: Intergen. transfers caused by private altruism may be suboptimal; and if intergen. transfers don't happen, maximising PV using market interest rate may not maximise ISWF.	(2): Whether or not equilib growth path is sustained depends on balance between rates of utility discounting and resource growth.	As (2), and: Only income transfers + lump sum resource taxes achieve maximin path; other policies achieve lower constant lifetime utility level

Aside from its avoidance of true intergenerational comparisons, there are several other moral and technical issues raised by defining SD as NDU. To avoid making this introduction too long, these will be discussed in the first substantive section of Chapter 2. But two other assumptions made for reasons of tractability in Chapters 2-4 are worth discussing briefly now. Firstly, the *infinite time horizon* used is unrealistic, since the Sun has a finite astronomical life. An infinite horizon leads to excessively gloomy results about the effect of finite resources on sustainability, as noted in Chapter 3 (though other assumptions, such as substitutability, arguably lead to excessively cheerful results). A finite horizon would be preferable, but again this would add too much to an already difficult modelling task – even though problems of assigning terminal values would not arise, since every asset will have zero value at the end of the time horizon, since this is also the end of the world. Secondly, all relationships in the models are *deterministic*, with the representative person having perfect foresight. This means that interesting questions which occur in stochastic and/or incomplete information models, such as stability and resilience (see for example Perrings 1989, Common and Perrings 1992 and Perrings 1993) are unfortunately ignored.

1.4 THE EXCLUSION OF RENEWABLE RESOURCES

We have already noted the predominant modern view, dating at least from WCED (1987), that it is the finiteness of renewable rather than non-renewable resources that poses a threat to the long run sustainability of humankind. This view heightens the connection between sustainability and environmental policy since renewable resources, being alive or forms of solar energy, generally tend to move around more than non-renewable resources. Renewables are therefore less likely to be owned and marketed, and more likely to be the source of environmental externalities. One of the earliest exponents of the significance of material resource flows in the economy now states that:

> "The most important scarcities, in the emergent environmentalist world view, are largely outside the market domain: soil fertility, clean fresh water, clean fresh air, unspoiled landscapes, climatic stability, biological diversity, biological nutrient cycling and environmental waste assimilative capacity." (Ayres 1993, p189)

But it is more difficult to study sustainability in economies based on renewable *substitutable* resources than on non-renewable, substitutable resources. To see this, define a general exhaustible resource as one whose stock $S(t)$ and rate of use $R(t)$ are related over time by the equation

$$\dot{S} = g(S) - R, \tag{1.1}$$

where $g(S)$ is the natural growth function. One can then distinguish at least four types of exhaustible resource:

(1) *Non-renewable* resources such as fossil fuels and minerals, where $g(S)=0$ on any realistic human timescale.

(2) *Living renewable* resources such as fish and forests, where $g(S)$ reflects the population growth of a biological species in a finite ecological niche. An example is $g(S)=\gamma S(\bar{S}-S)$ where $\bar{S}$ is some exogenously given carrying capacity (although this omits the notion of a minimum threshold below which a population is not viable).

(3) *Exponentially expanding* resources, where $g(S)=\Gamma S$, for some exogenous growth factor Γ, so that

$$\dot{S} = \Gamma S - R \tag{1.2}$$

This could be an asymptotic representation of $g(S)=\gamma S(\bar{S}-S)$ when $S \ll \bar{S}$. However, no natural resource could follow growth path (1.2) forever, since it is unbounded if $R=0$.

(4) *Replenishable resources* such as fresh water, where an exogenous supply $\bar{R}(t)$ fills a finite reservoir of size $\bar{S}$; for example, $g(S)=[(1-e^{S-\bar{S}})/(1-e^{-\bar{S}})]\bar{R}$. A variant on this would be a resource such as wind power where there is a maximum exogenous supply $\bar{R}(t)$, but no resource stock as such, so that resource extraction $R \leq \bar{R}$.

There is unfortunately no standard terminology for exhaustible resources which reflects this taxonomy. Many writers confine 'exhaustible resources' to mean (1), whereas it seems more logical to me to include (2) and (3) as well. Some writers take 'renewable resources' to mean (3) and (4) as well as (2), whereas others do not. But irrespective of terminology, the importance of this classification for my argument is as follows.

Firstly, it is obvious that the general dynamics of renewable, substitutable resources are less tractable than those of non-renewable resources, because renewables require at least two extra types of non-zero growth functions $g(S)$ ((2) and (4)) to be considered, with no reduction in complexity elsewhere. True, in *equilibrium* there will always be a simple rule available (set $R=g(S)$) which ensures sustainable *resource use*. But this says nothing about whether or not this produces a sustained *utility* path, whether or not either situation is part of a conventional dynamic optimisation, and if not, what policies would achieve sustained utility paths in economies with renewable resources which start from disequilibrium.

Secondly, although this is not shown by the above taxonomy, there are very few published models of the PV-optimal management of renewable resources which could be adapted for SD purposes. For even if there is no effective substitute for a renewable resource, it is ultimately the *utility* of the people who harvest the resource we are concerned about. But almost all dynamic optimisation models of non-renewable resources (such as Dasgupta and Heal 1979, Chapter 5, and Clark 1990) study the maximisation of the discounted harvest *income*. Since this ignores any changes in the marginal utility of income, such models are relevant to *sectoral* sustainability only (discussed further in Pezzey 1992, pp 347-350), which is not my purpose here. The only renewable resource model I know which maximises discounted utility rather than discounted income is Barbier and Markandya (1990). This reaches some interesting, albeit rather tentative, conclusions about phenomena such as PV-optimal self-extinction. So there would need to be further work on the PV-optimal, *macro*economic management of renewable resources, before one could analyse SD in such situations.

Thirdly, if the meaning of renewable resources is taken to include meaning (3), then renewables are indeed explored to some extent in this thesis. The resource in Chapter 4 is exponentially expanding. And the equation of motion for cake-eating with exogenous, exponential technical progress that is used in a special case of Chapter 2:

$$\dot{S} = -Ce^{-\tau t}, \quad \text{where } C \text{ is consumption,}$$

can be transformed, by writing $Se^{\tau t}=E$, the effective size of the resource base for consumption purposes, to

$$\dot{E} = \tau E - C.$$

This is equation (1.2) for an exponentially expanding resource, save for the distinction between consumption C and resource extraction R. Also, (1.2) is the continuous time version of the equation of motion that Mourmouras uses in his 1991 and 1993 papers, so his coverage of 'renewable' resources is no greater than mine.

I have now introduced my basic modelling framework. SD will be defined as the permanently non-declining instantaneous utility of a representative person with perfect foresight, in an infinite-horizon, deterministic economy based on non-renewable or exponentially expanding resources, which can be substituted for in both production and utility functions. I would stress again, in answer to critics of using this framework for modelling sustainability questions, its advantage in terms of comparability as well as tractability. Most of the neoclassical models, which for two decades have greatly influenced expert opinion on the significance of resource exhaustion and environmental degradation for the long-run economic future, have used the same assumptions. If SD modelling always avoids them, it will be in danger of remaining a separate sub-specialism of economics. Any of its pessimistic conclusions might then be dismissed by mainstream economists as being merely the product of their unconventionally pessimistic assumptions. This cannot be true for Chapters 2-4 below, the highlights of which are now briefly surveyed.

1.5 OVERVIEW OF CHAPTERS 2-4

Chapter 2, "Sustainability, Intergenerational Equity and Environmental Policy", first explores some more of the ethical and analytical foundations of the NDU criterion. It then models the depletion of a non-renewable resource which is privately owned and essential for consumption (either directly and with technical progress, or as an input to the economy's production function, with or without technical progress). The total resource stock is also an environmental amenity for everyone. The main contribution of the chapter is that it is the first systematic exploration of policies to achieve non-declining utility, and of the effect of environmental policy on the rise and fall of utility over time. (Most PV-optimal growth models are concerned only with what it takes to maximise the PV of utility, not with whether PV-optimal utility rises or falls).

Unfortunately, hard and fast results are difficult to come by. One broad conclusion to questions Q2 and Q3 is that a conventional, social PV-maximising environmental policy makes utility more likely to rise over time, but not necessarily to the point of achieving SD. Another is that in an economy with reversible capital investment but no technical progress, it seems impossible to find a practicable SD policy for the very long term: the policy will usually have to become a large subsidy if it is to provide a sufficient asymptotic incentive to encourage saving. This is despite the assumed endless substitutability of capital for resources. We also go some way towards answering question (Q5) on how SD policies could be justified in terms of personal preferences, by finding a case where collective action achieves SD at less PV cost per person than SD action by one person acting alone would cost.

Question Q4 on socially PV-optimal SD is also briefly addressed in Chapter 2, but only in a special case, and Q6 on sustainability indicators is ignored. Q7 is addressed, and we find an example where if the economy's initial consumption (and hence utility) level is given, it is possible for an exogenous *increase* in the initial resource stock to have the perverse effect of making an otherwise sustainable economy unsustainable. However, this example is based on living renewable resources, and it is hard to conceive of a similar example for non-renewable or exponentially expanding resources.

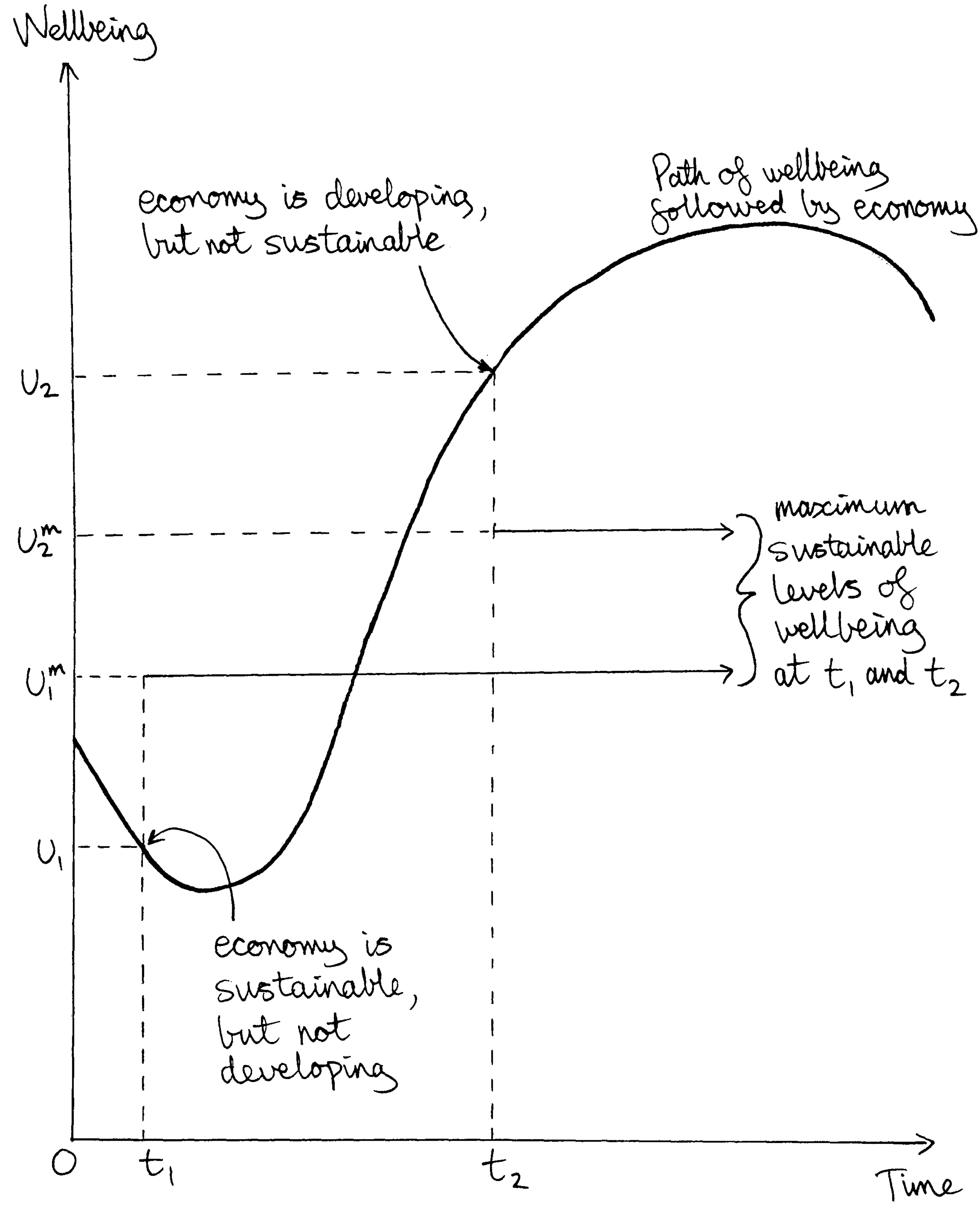
The first main part of **Chapter 3**, "The Optimal Sustainable Depletion of Non-renewable Resources", addresses Q6 on sustainability indicators, using a simple model with capital accumulation and non-renewable resource depletion, but no environmental externalities (so that the socially PV-optimal and free market development paths are the same) and no technical progress. As asserted earlier, non-declining aggregate wealth does not guarantee sustainability in this model, and so falls well short of the sustainability indicator we seek. The intuition behind this important result is as follows. Aggregate wealth is calculated by adding together the value of human-made capital and non-renewable resources. This addition must use *prices* to value resources relative to human-made capital. If relative resource prices are derived from market prices, then *even if full correction is made for all the conventional imperfections of those markets* (such as imperfect competition and incomplete current-generation property rights for natural resources), such prices do *not* reflect any sustainability constraint. In particular, at a crucial stage in an economy's development, the prices of non-renewable resources in perfect markets are likely to be too low for sustainability. Natural resource depletion is then undervalued and can be outweighed by investment in human-made capital, giving the 'false positive' message that wealth is increasing, when in fact the economy's consumption level is unsustainable.

The other main part of Chapter 3 takes up Q4 on PV-optimal SD (NDU at minimum PV cost), or 'opsustimality' as I have called it. The main contribution is to show that in general, SD requires a different development path to be followed *right from the start*, if the PV-optimal path is unsustained; and that the PV-optimal sustained path will comprise two phases. First there will be a phase of rising utility and no active policy intervention, as long as future intentions are credibly announced at time zero. Then there will be a phase of active policy intervention to achieve constant utility. But as in Chapter 2, such policy will again probably become impractical in the long run.

Chapter 4, "Concern for Sustainability in a Sexual World" again tackles Q5 on the justification of SD policy in terms of personal preferences. In Chapter 2, the environmental externality was used to justify collective rather than individual action to achieve SD. But what if, as in Chapter 3, there is no environmental externality? Does SD then have to be accepted *ex cathedra* as something that society ought to aim for, even though it appears to harm everyone's individual self-interest? If so, this would make SD unacceptable to many economists and politicians. But I believe that SD policy can respect individual interests, as long as one recognises an ever-present, but almost always ignored, *sexual externality*. I assume that sex results in people controlling only *half* of the bequests received by their offspring, as opposed to all of the bequests as happens if sex is absent. This then makes it inefficient, and perhaps impossible, for individual couples to achieve a sustained future for their descendants by their own actions alone. So provided that they value SD more than its cost in terms of PV, collective policy to achieve SD can be justified. Since bequests from parents to children are an essential part of reaching this result, Chapter 4 differs from Chapter 3 not just in having sexual rather than asexual agents, but also in modelling the representative person (or couple, in fact) as existing for a single, discrete time period, rather than for an infinity of continuous time.

Chapter 5 draws together the main conclusions of Part I.

Figure 1.1 Equivalence of sustainedness and sustainable development (SD)



CHAPTER 2

SUSTAINABLE DEVELOPMENT, INTERGENERATIONAL EQUITY AND ENVIRONMENTAL POLICY*

2.1 INTRODUCTION

> "The challenge of sustainable development is to promote ways of encouraging ... environmentally friendly economic activity, and of discouraging environmentally damaging activities. ... A key objective of environmental and sustainable development policy is to [ensure] that environmental costs and benefits are properly and fully taken into account in public and private sector decisions." (DOE 1994, p32)

These phrases, from a chapter entitled "Principles of Sustainable Development" in *Sustainable Development: The UK Strategy*, show how blurred the difference between sustainable development policy and environmental policy has become in the mind of at least one body politic in recent years. The chapter mentions 'sustainable development' and 'resources' only five times each, while 'environment' or 'environmental' appears over fifty times. The whole report often gives the impression that, at least for rich countries, sustainable development (SD) requires no more than an extension of environmental policies to tackle cumulative, global and/or irreversible problems such as ozone depletion, global warming and species loss. A good proportion of other relevant publications on SD by governments, consultants and academics would give the same impression.

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Yet, although there is much overlap between a policy to take account of cumulative environmental problems and a policy to achieve SD, environmental policies and SD policies are distinct, at least in principle. There may be an embarrassing plethora of definitions of SD (Pezzey 1989), but it is clear that the ultimate goal of most of them is not that of environmental policy, which (at least in theory) is to achieve *social PV-optimality*, i.e. the maximisation of the present discounted value (PV) of utility over time, taking into account the effect of environmental degradation on per capita PV.

Rather, SD primarily aims to achieve a particular form of intergenerational equity: making sure that future generations have similar levels of well-being or utility as enjoyed by those before them, or at least the opportunity to attain such levels. (Note that this aims for more than the mere survival of future generations.) I define SD here as *non-declining utility (NDU) forever*. In particular, an entire development path is 'sustained' if the utility of a representative person in the economy never declines along the entire path. Basic details of this definition, and the tractability and comparability reasons why it was chosen in preference to non-declining lifetime welfare in an overlapping generations framework, were discussed at length in Section 1.3 of Chapter 1, and further aspects of NDU will be discussed shortly.

But it is immediately obvious that as long as SD entails any form of intergenerational equity that is not included in social PV, achieving an SD path will generally require a different degree, or even a different type, of policy intervention than achieving an environmentally correct path does. If the current generation does not care much about the future, the future may be quite dismal, even though cumulative environmental problems have been fully 'taken into account', and even though this makes the future somewhat less dismal than it would otherwise be. The danger in allowing sustainability and environmentalism to merge in the public mind is therefore that 'sustainable development' or 'sustainability' may become just a convenient relabelling of existing environmental policy goals, rather than a fundamental reappraisal of society's ethical attitudes towards the future.

It may also lead to technical oversights. *If* capital accumulation and technical progress can continue to substitute indefinitely for environmental resources – which is an assumption I make throughout this thesis, in spite of considerable doubts as to whether it is true in practice – then it turns out that increasing an economy's overall savings ratio or rate of technical progress may be a more important way of achieving SD than direct incentives to conserve environmental resources. Yet savings policy receives scant attention in DOE (1994), or in most of the SD literature.

The main purpose of this chapter is to illustrate the technical differences between SD and environmental policies. This is done by analysing the effect of an NDU constraint on two neoclassical growth models, based on Krautkraemer (1985), where population is large, uniform and constant, and depletion of a privately-owned non-renewable resource with social amenity value causes an environmental externality. Many alternative models could have been chosen, and those used here are meant to explore sustainability problems rather than answer them definitively. To explain what is new here, I first need to define SD and PV-optimality terminology more precisely, using **Figure 2.1** for illustration. $U(t)$, the utility of an infinitely-lived person representative of the whole population over all time t (which runs continuously from 0 to ∞), is also used in the Figure to represent an entire development path of an economy (including its resource and capital stocks) over all time. $U^{\mu}(t)$ is the *free market* path[1] which is chosen by an agent who maximises her PV while ignoring the social amenity cost of her personal resource depletion. $\widetilde{U}(t)$, the *socially PV-optimal* path, is what she would follow if this cost were to be internalised.[2] The required environmental policy is $\widetilde{\Phi}(t)$, a vector of tax incentives with which the government induces individual agents to follow $\widetilde{U}$ rather than U^{μ}.

$\widetilde{U}(t)$ may or may not be a member of $\{U^{\dagger}(t)\}$, the set of all *sustained* or *SD* paths.[3] These are formally defined as paths with NDU forever, i.e. for which $U(t_2) \geq U(t_1)$ for any t_1, t_2 with $t_2 > t_1 \geq 0$. If utility is differentiable, a sustained path is one where $\dot{U}(t) \geq 0$ for all $t(\geq 0)$, and for brevity we stay with this differentiable definition (the non-differentiable equivalent is easy to infer). An *unsustained* path is then one which is declining ($\dot{U} < 0$) at some time. Some writers use what I would call asymptotic SD ($\lim_{t\to\infty} U$ is greater than some minimum level) as a definition of SD itself, but I do not, since policymakers care about the immediate as well as the distant future.

As noted in Chapter 1, a path is then *sustainable at time t* if a particular sustained path exists that starts from the current capital and resource stocks and the current utility level at t; otherwise, it is *unsustainable at t*. I assume

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1. The emphasis is more on 'free' (from government intervention) than on 'market', since markets formally do not exist in some of the economies we consider. Where they do exist, they are assumed to be competitive.

2. I assume here that the necessary conditions hold for the existence and uniqueness of the PV-optimal path.

3. When they both are used as qualititative descriptions of development paths, *sustainedness* and *sustainable development (SD)* continue to be treated as synonymous terms in this Chapter. However, later we also talk about sustainedness as if it might be quantified (as in 'to improve sustainedness'), in which case SD does not seem quite such an apt term.

that the economy is such that some sustained path does exist.[4]

Two sustained paths are of particular interest. I assume that SD policy has no desire for the hair shirt for its own sake, and ideally aims for the sustained path which has the highest PV, which I call the *opsustimal* path and denote as $\tilde{U}^{\dagger}(t)$. However, the opsustimal path is rarely easy to characterise, and only in Chapter 3 where environmental externalities are absent can much progress be made. The sustained path which maximises $[\min_{\text{all } t\geq 0} U(t)]$ is called the (initial) *maximin* path, labelled U_0^m.[5] Except for trick cases, this is also the *maximum constant utility* path. As for policies, $\{\Phi^{\dagger}(t)\}$ is the set of all SD policies which convert $U^{\mu}(t)$ to paths in $\{U^{\dagger}(t)\}$. Of these policies, $\tilde{\Phi}^{\dagger}(t)$ and $\Phi_0^m(t)$ respectively convert $U^{\mu}(t)$ to the opsustimal path $\tilde{U}^{\dagger}(t)$ and the initial maximin path $U_0^m(t)$. $\hat{U}(t)$ is the general policy path which a private PV-optimising agent follows in response to a general policy $\Phi(t)$.

What I attempt to do here, by modelling the depletion of a non-renewable resource with social amenity value, is to compare the free market path U^{μ}, the socially PV-optimal path $\tilde{U}$, a sustained path $U^{\dagger}$, and the environmental and SD policies $\tilde{\Phi}$ and $\Phi^{\dagger}$ which respectively transform the first path into $\tilde{U}$ and $U^{\dagger}$. Some of the questions which will be addressed are: When does environmental policy convert an unsustained free market path ($\dot{U}^{\mu}<0$ sometime) into a sustained one ($\dot{\tilde{U}}\geq 0$ always)? If it does not ($\dot{\tilde{U}}<0$

4. Cass and Mitra (1979) gave the general conditions for sustained, strictly positive consumption to exist, but this says little about the existence of non-declining utility in the presence of an environmental amenity. Discovering the general conditions for this latter case (which is relevant to this chapter) remains for further work.

5. The qualifier 'initial' distinguishes U_0^m from $U_{t'}^m$, the *current* maximin path that maximises $[\min_{\text{all } t\geq t'} U(t)]$, which is explored further in Chapter 3 and (although they did not call it such) by Bishop and Woodward (1994).

sometime), what is the minimum that SD then costs (what is $PV(\widetilde{U})-PV(\widetilde{U}^{\dagger})$)? What are the best instruments to use as part of environmental policy $\widetilde{\Phi}$ and SD policy $\Phi^{\dagger}$?

It is this formal comparison of SD and environmental policies which is new to the literatures on intergenerational equity and/or dynamic externalities in resource economies. For although Dasgupta and Heal (1974) and Stiglitz (1974) effectively showed that the PV-optimal path of a capital-resource economy with no externalities will be unsustained if respectively the marginal productivity of capital or the rate of technical progress is too low, they did not treat this as a policy problem. Solow (1974) showed the conditions for a non-zero constant utility path U_0^m to exist in a Cobb-Douglas version of the economy, and recognised (p36) that policy intervention would be needed to achieve it, but he did not analyse what form the policy $\Phi_0^m(t)$ should take. Hartwick (1977) derived his famous rule of investing all resource rents in order to achieve constant utility. But like Solow, he did not identify any policy which could motivate PV-maximising agents to achieve this level of investment,[6] and policy analysis is rare in the literature he inspired.

One exception is Dasgupta and Heal (1979, p291), who showed how a constant income tax rate can achieve constant utility, but only if the government is the sole investor. This seems of little relevance to a market economy where the bulk of resource depletion and investment is controlled by private agents.[7] Becker (1982) went further, and described maximin

6. As discussed in Section 2.2, it may well still be rational for such agents to support a government which pursues an SD policy.

7. For example, during the 1980s the UK private sector typically accounted for more than three-quarters of gross domestic fixed capital formation, and more than two

policies $\Phi_0^m(t)$ operating on both capital and environmental markets in an economy with externalities, but he discussed neither environmental nor SD policies. Schulze (1974), Sweeney (1977) and Krautkraemer (1985) all effectively showed how an environmental policy which internalises an externality from non-renewable resource depletion can shift resource extraction from the present to the future. These papers thus suggest that environmental policy makes utility less likely to decline over time, but none of them considered intergenerational equity. In this paper I adapt Krautkraemer's models for analysing SD, because they explicitly model the socially PV-optimal utility path $\tilde{U}$ in general equilibrium (i.e. in a whole economy, albeit a single-sector one, where the resource demand is endogenous), unlike the Schulze and Sweeney models which are partial equilibrium. In the recent sustainability literature, Van Den Bergh and Nijkamp (1991) and Klaassen and Opschoor (1991) surveyed techniques of modelling SD in the presence of environmental values, but did not develop the NDU criterion as such. Finally, in one of the overlapping generations models noted in Chapter 1, Howarth and Norgaard (1992) developed a model with cumulative pollution rather than resource degradation as the source of the externality.[8] They defined SD as non-declining *generational* utility, and described the environmental policy and showed the effect of the utility discount rate on PV-optimal utility growth $\dot{\tilde{U}}$, but they did not discuss SD policies per se.

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thirds of total investment (defined as fixed capital formation and all expenditure on human capital, i.e. education) (GSS 1993, p112).

8. Cumulative pollution and non-renewable resource depletion are analytically equivalent if there is a strict upper threshold for pollution, beyond which the economy collapses; but Howarth and Norgaard do not include such a threshold.

We could now proceed to the formal analysis of our two growth models. However, in view of continuing controversy about the NDU constraint, it is worth using Section 2.2 to set out some arguments for studying NDU as a goal of public policy, and some alternative criteria of intergenerational equity, all within the context of continuous time, infinitely lived, representative person models. Section 2.3 then analyses SD and environmental policies in a general model of cake-eating with technical progress, and in two special cases. Section 2.4 extends the analysis to a productive economy with capital-resource substitution, where measures to increase saving can then be distinguished from measures to conserve resources. Section 2.5 investigates some special effects caused by having a fixing starting level of consumption, and hence utility. Section 2.6 concludes.

2.2 SD AS NON-DECLINING UTILITY (NDU)

I do not claim here that NDU is the 'right' or 'best' definition of SD, or results in a 'better' allocation of wellbeing over time than unconstrained PV-optimality. I aim merely to show that it deserves proper economic analysis. My approach towards NDU closely parallels that of Solow (1974) towards maximin utility. Both he and I are saying in effect: "Here is an interesting criterion of intergenerational equity, proposed or popularised by an influential source (in our case WCED 1987, in his case Rawls 1971) and much debated since. Let us analyse it with formal economic techniques, thereby learning more about it both normatively (What are its strengths and weaknesses from a political or philosophical point of view?) and positively (Is it feasible? When it is, how can it be achieved?). In so doing, we hope to advance the debate about the choice of criterion; but we do not say that this is *the* correct choice for society to make."

As noted in Chapter 1, there are technical reasons for choosing NDU in preference to three other criteria of intergenerational equity. It has been chosen in preference to non-declining lifetime welfare in an overlapping generations model, for reasons of tractability. It has been chosen in preference to a condition of non-declining natural resources ('natural capital'), in order to avoid relying on assumptions that resources are non-substitutable beyond some threshold. It has been chosen in preference to non-declining aggregate wealth because, as explained in Chapter 3, the latter does not guarantee NDU.

NDU has also been chosen for study because of its sheer popularity. Once one has translated NDU into popular language, one finds an enormous volume of statements in its favour in academics' and (especially) policymakers' pronouncements on intergenerational equity. NDU is implicit in at least half of the dozens of written definitions of sustainability and SD collected in Pezzey (1989), and five years later governments far and wide have proclaimed their commitment to achieving SD. The contrast with the maximin criterion is striking. Any proposal to ban future increases in human wellbeing, which is the 'poverty trap' implied by the maximin (Solow 1974), would attract little support from current governments or their electorates. This is surely because most of them expect to be around to enjoy some of the future, and do not consider a rising utility path to be automatically unfair to the current generation.

To go further than political economy, and 'justify' NDU as an ethically valid criterion for intergenerational equity, involves making a choice between different value systems, and in my view the debate about this choice can never be resolved. It can be clarified in the manner of Asheim (1991), who effectively reduced NDU to yet more fundamental principles.

He showed that any 'just' consumption paths must have NDU, where a just path is one which cannot be overtaken by any other path in terms of both total *un*discounted consumption, and the equality of the allocation of consumption over time. But proponents of PV-optimality could point out that Asheim's definition of justice is almost bound to conflict with a discounted criterion like PV, but that does not prove the former to be ethically superior.

Indeed, some 'PV-optimalists' accept that intergenerational equity is a valid concern, but claim that the correct choice of (utility) discount rate in the PV calculation is one way of taking care of it (Beckerman 1994, p199). This does happen in some special cases of models with exponential technical progress in this chapter, where the PV-optimal path is sustained as long as the discount rate is low enough. In other models with sub-exponential or zero technical progress, however, for any positive discount rate the PV-optimal path declines to a level of misery, even though unbounded utility growth is feasible. Such an outcome runs contrary to many, maybe most, people's notions of intergenerational equity. The PV-optimalist response would probably be that technical progress will never be subexponential, but this is an empirical belief that many do not share.

A related PV-optimalist criticism is that an NDU policy does not respect individual preferences, whereas PV-optimality does, since individuals do generally discount the future. How can it be logical for individuals (say) to have a utility discount rate which causes an unsustained PV-optimal path, and yet vote for a government which enacts policies which effectively lower this discount rate?[9] Why do they not instead change their personal discount

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9. In this we assume that environmental policy has already internalised the resource's social amenity value, but that the resulting utility path is unsustained.

rate to achieve individual sustainedness? Similar questions arise with a maximin criterion, although they remain largely unaddressed in the literature.

One answer is to assume that "the Economic Man and the Citizen are for all intents and purposes two different individuals" (Marglin 1963, p98), i.e. that individual and social preferences are simply separate. A more satisfying answer would follow from Conjecture 2.1 in Section 2.3.2 below. This is based on both intuition and special cases of cake-eating economies there and in Chapter 4, and speculates that whenever there are externalities from cumulative resource depletion in the economy, and when the socially PV-optimal path is unsustained, *SD is partly a collective good.* That is, because of the harmful effect of total resource depletion on an individual agent's utility, it will cost her more PV to achieve SD through her individual resource conservation effort (assuming that no one else makes any effort to achieve SD) than the PV cost to her of a collective SD policy. We refer to this later as the cost of *individual* SD being greater than the cost of *social* SD.

Two further supposed drawbacks of an NDU constraint turn out to be illusory, provided that the economy is 'productive' as defined by Asheim (1991, pp. 357-8). This essentially means that an SD path exists, and that one can always transform an unsustained path into an SD one by saving before any time of declining utility, and transferring the saving to the later time to 'iron out' the decline. Consider **Figure 2.2**, which plots alternative feasible paths of utility over time, none of which are necessarily optimal in any way. It might seem that an NDU constraint would choose the non-declining path P1 in preference to the everywhere-superior, but partly-declining path P2. But if the economy is productive, one can modify P2 by following some constant utility segment such as the dashed line *abc* (by saving between a and b and dissaving between b and c) without changing the rest of the path. PV-optimal SD would then prefer the modified path P2′ to P1, thus avoiding a clearly inferior choice of development path.

A similarly mistaken argument is that NDU would prefer P2′ to the initially-declining path P3 (which represents some initial investment phase which is highly productive later on), even though P3 might have much higher PV than P2′. Again, if the economy is productive, a constant utility segment *de* must be feasible which 'irons out' the dip at the start of P3 to produce the sustained alternative P3′ with (say) much higher PV than P2′.[10]

The only genuine drawbacks of an NDU constraint are firstly, that it will of course reduce PV if it is binding. Indeed, one could define a (social) *cost of SD* in this case as PV(P3) − PV(P3′),[11] and choose the SD path P3′ in preference to P3 only if this cost is outweighed by some finite *value of SD* arising from the fact that P3′ has NDU. It would therefore be helpful to find out more about the PV cost of SD, but unfortunately it can be calculated only in the first special case of the cake-eating model. Secondly, NDU may also force a drop to a lower initial utility level (shown by the drop from f on P3 to d on P3′ in Figure 2.2). This matters in a more realistic model of policy-making where, as discussed in Section 2.5, an initial utility level is inherited from history, and where rapid utility declines below this level cause adjustment costs.

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10. I thank Geir Asheim for pointing this out to me.

11. It is only *a* cost of SD, because a natural definition of *the* cost would be $\mathrm{PV}(\widetilde{U}) - \mathrm{PV}(\widetilde{U}^{\dagger})$, where $\widetilde{U}^{\dagger}$ is the opsustimal path. This cost would be the Lagrange multiplier associated with the NDU constraint, but we will see in Section 2.3 that this cannot be calculated using general optimal control techniques.

Two types of alternatives to NDU exist as criteria of SD. The first type still treats SD as a side constraint on PV-optimality, but changes the variable that must be non-declining over time t from utility $U(t)$ to one of the following: $PV_t(\widetilde{U})$, the maximum PV available from time t onwards (this might allow P3 to be judged both sustained and better than P3′); $PV_t(\widetilde{U}^\dagger)$, the PV of the opsustimal path from t onwards; or just U^m_t, the maximum constant utility level attainable from t onwards (whether or not measured on a PV-optimal path).[12] Or, one could look for the maximin of these variables, rather than just require that they are non-declining.[13]

The second type of alternative criteria follows the suggestion of Broome (1992, p40) and extends the present value calculation to include some finite weight for intergenerational equity, thus creating a more general form of intergenerational social welfare function (ISWF).[14] The simplest option would be to follow the previous paragraph but one and add a discrete 'value of SD' into the PV integral; or one could add in a continuous function of the equality of utility among different times; or one could add some constant multiplied by the asymptotic utility level, as suggested by Beltratti,

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12. As noted in Chapter 1, Riley (1980) investigated some of the properties of a non-declining $PV(\widetilde{U})$ criterion, under fairly restrictive technological assumptions. Non-declining U^m_t is the 'resourcist' criterion in Bishop and Woodward (1994), who focused on the opportunity set bequeathed to the future, rather than the level of welfare achieved, which can be affected by changes in the efficiency of resource use as well as in opportunity. In my models below the level of efficiency is unchanging.

13. The *maximin PV* path, which maximises $\min_{s\geq t}[\int_s^\infty U(x)e^{-\delta x}dx)]$, was investigated in great detail (in its discrete time version $\min_{s\geq t}[\Sigma_{i=s}^\infty U_i/(1+\delta)^{i-s}]$) by Asheim (1988). He showed that it allows for initial utility growth when investment is very productive, thus avoiding the maximin 'poverty trap' mentioned above.

14. See also the discussion of ISWFs in Toman, Pezzey and Krautkraemer (1995, forthcoming).

Chichilnisky and Heal (1993); or one could abandon discounting in the PV integral altogether, and substitute inequality aversion as the intertemporal weighting factor, as in a novel approach by Collard (1994). Any of these options would tend to modify the PV-optimal utility path to one with less tendency to decline in the long run. For the sake of political reality alone there is much to be said in favour of these 'weighing' or 'valuing' criteria of intergenerational equity. But the technical properties of all the above alternative criteria are complex and, apart from in the papers noted here, mostly remain for further research. Any final choice between them will certainly be helped by knowing more about of the basic NDU criterion, which is our purpose here.

2.3 CAKE-EATING PLUS TECHNICAL PROGRESS

2.3.1 The general cake-eating model: comparison of SD and environmental policies

A slight generalisation of Krautkraemer's first model is as follows. For the moment, the analysis is from the viewpoint of society as a whole, but when we come to analyse the policy path later we will also take the viewpoint of one of the millions of identical agents that comprise society. The social problem is:

$$\max_{\{R(t)\}} \int_0^\infty e^{-\delta t} U[C(t),S(t)]dt \quad =: \; PV, \text{ the present discounted value of utility, where } \delta > 0 \text{ is the utility discount rate} \tag{2.1}$$

subject to:

$$C(t) = A(t)R(t) = -A(t)\dot{S}(t); \quad \dot{A}(t) > 0, \text{ all } t; \tag{2.2}$$

$$S(0) = S_0 > 0; \quad R(t),\ S(t) \geq 0, \text{ all } t; \tag{2.3}$$

where U, C, R and S are instantaneous utility, consumption, resource (depletion) flow and resource stock respectively. $A(t)$ is an exogenous technical productivity factor ($=e^{\tau t}$ in Krautkraemer) and we assume by suitable choice of units that $A(0)=1$.[15] The arguments in the utility function $U(C,S)$ respectively capture the materialistic value of consumption, and the amenity value of the 'environment', i.e. the total resource stock. U is assumed to be strictly increasing, strictly concave and twice differentiable in both of its arguments, with the marginal amenity value rising as consumption rises, and the marginal utility of consumption being unbounded as consumption falls to zero, i.e.

$$U_C>0 \text{ and } U_{CC}<0; \quad U_S>0 \text{ and } U_{SS}<0; \quad U_{CS}>0; \tag{2.4}$$

$$\text{and} \qquad U_C \to \infty \quad \text{as} \quad C \to 0. \tag{2.5}$$

$U_{CS}>0$ in (2.4) means that, as in Krautkraemer, a higher standard of living increases the marginal valuation of the environment. The solution of (2.1)-(2.5), fully characterised in Krautkraemer for $A=e^{\tau t}$, is the socially PV-optimal path $\widetilde{U}(t)$. The path that also satisfies the NDU (i.e. SD, i.e. sustainedness) constraint

$$\dot{U}(t) \equiv \dot{C}U_C - RU_S = \dot{C}U_C - (C/A)U_S \geq 0, \quad \text{for all } t \tag{2.6}$$

is the opsustimal (PV-optimal sustained) path $\widetilde{U}^{\dagger}(t)$. A constant consumption ($C>0$, $\dot{C}=0$) path is shown later to be not PV-optimal. Nor is it sustained, by (2.4) and (2.6). Strictly rising consumption is necessary for SD, both here and in the capital-resource economies considered in Section 2.4.

Can the constraint (2.6) give useful results when added to the usual optimal control framework? The undiscounted Hamiltonian of (2.1)-(2.6)

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15. We do not consider $\dot{A} = 0$ (no technical progress) here because pure cake-eating is unsustainable; but we do consider it in Section 2.4, because pure capital-resource substitution can be sustainable.

is

$$H = U(C,S) - \pi_S(t)R + \lambda_R(t)R + \lambda_S(t)S + \lambda_U(t)\dot{U}$$

which from (2.2) is

$$H = U(C,S) - [\pi_S(t) - \lambda_R]C/A + \lambda_S S + \lambda_U(t)[\dot{C}U_C - (C/A)U_S]$$

where $\pi_S(t)$ is the undiscounted shadow price of the resource stock in terms of utility and λ_R, λ_S and λ_U are Lagrange multipliers belonging to the non-negativity constraints (Chiang 1992, p279). Using the maximum principle, the first order and complementary slackness conditions are respectively

$$\partial H/\partial C = 0, \quad \partial H/\partial S = -\dot{\pi}_S + \delta\pi_S$$

and $\quad \lambda_R,\ \lambda_S,\ \lambda_U \geq 0, \quad \lambda_R R,\ \lambda_S S,\ \lambda_U \dot{U} = 0.$

As in Krautkraemer, the fact (from (2.5)) that the marginal utility of consumption is unbounded as consumption approaches zero means that PV-optimal R and S must be strictly positive. But there is no similar marginal condition to prevent the SD constraint binding ($\dot{U}=0$), so we cannot ignore λ_U in deriving the first order conditions for the PV-optimal path, which are

$$U_C - \pi_S/A + \lambda_U[\dot{C}U_{CC} - (U_S + CU_{CS})/A] = 0 \tag{2.7}$$

$$U_S + \lambda_U[\dot{C}U_{CS} - (C/A)U_{SS}] = -\dot{\pi}_S + \delta\pi_S \tag{2.8}$$

The next step would normally be to differentiate (2.7) with respect to time and then substitute for $\dot{\pi}_S$ from (2.8), but this leads to impenetrable algebra involving third derivatives when $\lambda_U > 0$. In any case, the solution will be different when $\lambda_U = 0$, so the complete solution path will generally have distinct phases. This severely limits our ability to describe the opsustimal path, except when exact solutions are available in steady state cases, and we have to adopt a more ad hoc approach. We drop SD as a formal constraint with a characterisable Lagrange multiplier, and instead study what interventions (if any) are both feasible and necessary to make $\dot{U}$ zero, when otherwise it would be negative. From (2.7) and (2.8), the socially PV-optimal growth rate of consumption can be shown (see Appendix 2.1) to be a slight variant on Krautkraemer's equation (10):

$$\dot{\widetilde{C}} = [\dot{A}/A - \delta + (\widetilde{U}_S - \widetilde{C}\widetilde{U}_{CS})/\widetilde{U}_C A]\widetilde{C}/\eta(\widetilde{C}) \tag{2.9}$$

where $\eta(C) = -CU_{CC}/U_C$ is the elasticity of the marginal utility of consumption. From his Lemma A1, with $\pi_S = U_C A$, the transversality condition is

$$\lim|_{t\to\infty}(\widetilde{U}_C A e^{-\delta t}\widetilde{S}) = 0. \tag{2.10}$$

The corresponding utility path, here and later, is derived from inserting the consumption path (here (2.9)) into (2.6):

$$\dot{\widetilde{U}} = [(\dot{A}/A - \delta)\widetilde{U}_C + \widetilde{U}_S/A - (\widetilde{C}\widetilde{U}_{CS} + \eta\widetilde{U}_S)/A]\widetilde{C}/\eta(\widetilde{C}) \tag{2.11}$$

The $\widetilde{U}_S/A$ term is isolated in (2.11) to aid comparison with later variants of this equation, and it shows that if $U_S > CU_{CS} + \eta U_S$ for all C and S, then the existence of the resource amenity *improves the sustainedness* of the PV-optimal path.[16] By this we simply mean that there is a positive impact effect on the expression for the rate of change of utility; and to *worsen sustainedness* means to have a negative impact effect on the expression for the rate of change of utility. An 'improvement in sustainedness' is thus a useful but slightly deceptive shorthand, since it tends to suggest that some measure of the overall sustainedness of a path is increased. But (2.6) defines sustainedness only as a discrete condition, not as a continuous number. One could perhaps say that the sustainedness of any $U(t)$ path is

16. Note that this is a stricter condition than Krautkraemer's $U_S > CU_{CS}$ condition for the resource amenity to lead to higher PV-optimal *consumption* growth; the extra ηU_S term represents the direct effect on utility of a degrading environment.

improved if an integral like $\int_0^t [\min(\dot{U},0)]dt$ is increased for all t, but I have not investigated any such formal measure. So we know little about what an 'improvement' in sustainedness means for $\widetilde{U}$ at any particular time. We do not even know that the initial level of $\widetilde{U}$ is lower on an 'improved' path. This is because the improvement in sustainedness may be the result of a policy which converts a non-PV-optimal path into a PV-optimal one which may strictly dominate the former at all times.

Assuming that $U_S > CU_{CS} + \eta U_S$, (2.11) shows that technical productivity A has an ambiguous influence on sustainedness. An increase in the rate of progress $\dot{A}/A$ improves sustainedness by making people want to defer consumption to the future, but a higher level of productivity A worsens sustainedness by reducing the resource flow needed for a given consumption level, and hence reducing the amenity motive for conserving resources.

We now calculate the *policy path* of the economy, where we consider the economy from the viewpoint of the private, PV-maximising agent, as influenced by government policy. We assume that, in a market economy, policymakers can influence private agents' consumption and resource depletion only by fiscal incentives, rather than by direct regulatory control. Given the immortal, representative person format here, it is impossible to model the intergenerational asset transfers suggested by Howarth and Norgaard (1992). However, since future generations have little legal status, one could argue that such transfers usually amount to fiscal or regulatory control of the current generation's consumption and resource depletion anyway. The most obvious incentives are:

- a revenue-neutral *consumption tax* (or 'fee') charged at a rate of $\phi_C(t)$ (which, since $C=AR$, has the same potential as a resource depletion tax);
- a revenue-neutral *resource stock tax* charged at a specific rate $\phi_S(t)$, which we expect to be negative, representing a subsidy. (Since a stock is usually harder to measure economically than a flow, ϕ_S is probably the less practical instrument of the two.)

Since the rate of technical progress is assumed to be exogenous, we cannot model policies to increase it, but if such policies did exist, their likely benefit will be evident in what follows. Revenue-neutrality with ϕ_C and ϕ_S is achieved by lump sum rebates Ω. If the incentive is a subsidy, Ω is a lump sum tax, which would pose severe political problems in the real world where people are far from economically identical, as the recent British experience of lump-sum local taxation (the 'poll tax') has shown.

The PV-optimality problem of a private agent with an initial resource stock s_0 and subject to policy intervention which affects her budget constraint is

$$\underset{\{c(t)\}}{\text{MAX}} \int_0^\infty e^{-\delta t}u[c(t),S(t)]dt \quad \text{s.t.} \quad \int_0^\infty [c(1+\phi_C)/A+\phi_S s-\omega]dt \leq s_0$$

where u, s, c $(=-A\dot{s})$ and ω are respectively *per capita* utility, resource stock, consumption, resource depletion and lump sum rebate. Each agent externalises the impact of her resource depletion on the total resource stock S (and hence on its amenity value u_S) when computing her utility, but treats her private resource stock s as under her control when computing her budget constraint. For brevity's sake we can represent the amenity externality simply by using the society-level variables U, C and S in the undiscounted Hamiltonian

$$H = U(C,S)-\pi_S(t)[C(1+\phi_C)/A+\phi_S S-\Omega]$$

and omitting any terms in U_S which would normally appear. We then do not need to consider the individual as such any further: a privately PV-optimal resource depletion path will satisfy the first order conditions:

$$\partial H/\partial C = U_C - \pi_S(1+\phi_C)/A = 0 \tag{2.12}$$

$$\partial H/\partial S = -\dot{\pi}_S + \delta\pi_S = -\pi_S\phi_S \tag{2.13}$$

By taking the time derivative of (2.12) and substituting from (2.13), it can be shown (see Appendix 2.2) that consumption and utility on the path followed by an agent maximising her PV in response to policies $\phi_C(t)$, $\phi_S(t)$ are:

$$\dot{\hat{C}} = [\dot{A}/A - \delta + \Phi - \hat{C}\hat{U}_{CS}/\hat{U}_C A]\hat{C}/\eta \tag{2.14}$$

and $$\dot{\hat{U}} = [(\dot{A}/A - \delta + \Phi)\hat{U}_C - (\hat{C}\hat{U}_{CS} + \eta\hat{U}_S)/A]\hat{C}/\eta, \tag{2.15}$$

where $$\Phi(t) := -\dot{\phi}_C/(1+\phi_C) - \phi_S \tag{2.16}$$

measures the effective overall strength of policy intervention at any time. (2.14) and (2.15) reduce to the free market solution if there is no policy intervention ($\Phi = 0$):

$$\dot{C}^\mu = [\dot{A}/A - \delta - C^\mu U^\mu_{CS}/U^\mu_C A]C^\mu/\eta \tag{2.17}$$

$$\dot{U}^\mu = [(\dot{A}/A - \delta)U^\mu_C - (C^\mu U^\mu_{CS} + \eta U^\mu_S)/A]C^\mu/\eta \tag{2.18}$$

Examining first the free market solution, from (2.4) the $-(C^\mu U^\mu_{CS} + \eta U^\mu_S)$ term in (2.18) is negative. So whenever the technical progress rate $\dot{A}/A$ falls below the discount rate δ (which is eventually bound to happen if progress is less than exponential), the private outcome is unsustained. The converse is not necessarily true, though. Another feature of (2.18) is worth highlighting as:

REMARK 2.1: *The more highly the environment is valued, the worse sustainedness will be on the free market path of development.*

By this we simply mean that the larger are the $\partial/\partial S$ terms in (2.18), the more negative the utility increase will be, other things being equal. As noted above, this is a somewhat vague result. Other things are not equal, and other endogenous variables in (2.18) will be affected by any change in the $\partial/\partial S$ terms, so that $\dot{U}^{\mu}$ may not decrease at every point in time. I have not investigated what happens to any formal, continuous measure of the sustainedness of the overall development path. However, it is interesting to observe that environmental concern in (2.18) worsens sustainedness for two distinct reasons. The $-C^{\mu}U^{\mu}_{CS}$ term (<0, by assumption) shows the 'active' (or what might be called the 'live now, pay later') effect. Because agents value consumption more highly when the non-renewable 'environment' S is less degraded, they shift their consumption and hence resource depletion towards the present, making the future even worse by comparison. The $-\eta U^{\mu}_{S}$ term shows the 'passive' effect that resource depletion inevitably reduces each agent's amenity value.

Comparing (2.18) and (2.11) shows that the *environmental policy* adds $U_S C/A\eta$ (>0) to $\dot{U}$, and thus improves sustainedness. This is also worth highlighting as:

REMARK 2.2: *An environmental policy in a market economy with non-renewable resource amenity value will 'improve sustainedness' (so if the free market path is sustained, the socially PV-optimal one certainly is); and may achieve a sustained path as a side-effect, but this is not guaranteed. A socially PV-optimal path may be unsustained, in which case additional policy intervention is needed to achieve sustainedness.*

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Comparing the socially PV-optimal and policy paths (2.11) and (2.15) shows that the required environmental policy must at all times be

$$\tilde{\Phi} = -[\dot{\tilde{\phi}}_C/(1+\tilde{\phi}_C)+\tilde{\phi}_S] = \tilde{U}_S/A\tilde{U}_C > 0$$

So the environmental policy often ends up involving *subsidies*, with the associated financing problems in practice already noted above. If the sole policy instrument is a resource stock 'tax', this is always true (since $\tilde{\phi}_C=0$ $\Rightarrow \tilde{\phi}_S<0$, i.e. a stock subsidy). A similar result holds if the sole tax is on consumption, but for this we first need:

LEMMA 2.1: *If $-\dot{\phi}/(1+\phi)$ is always positive and bounded away from zero, then $\lim_{t\to\infty}\phi = -1$.*

PROOF: The subsidy rate has to be less than 100% at any one time, i.e. $\phi>-1$, or else an individual's desired consumption would be unbounded. This in turn means that $\dot{\phi}<0$, to get the right sign of $-\dot{\phi}/(1+\phi)$. So $\lim_{t\to\infty}\phi=-1+z$ for some finite $z\geq 0$, and $\lim_{t\to\infty}\dot{\phi}=0$. But then $\lim_{t\to\infty}[-\dot{\phi}/(1+\phi)]=0/z$, and $\lim_{t\to\infty}[-\dot{\phi}/(1+\phi)]$ is positive by assumption. Therefore, z cannot be non-zero. ||

Applying this to a consumption tax $\tilde{\phi}_C$ on its own (i.e. with $\tilde{\phi}_S=0$), if $\lim_{t\to\infty}\tilde{U}_S/A\tilde{U}_C=0$, the tax must approach a constant value (possibly, but not necessarily negative); whereas if $\lim_{t\to\infty}\tilde{U}_S/A\tilde{U}_C>0$, the tax must approach -1, i.e. approach a 100% subsidy on consumption.

An *SD policy* $\Phi^{\dagger}(t)$ will be any policy which attains $\dot{\hat{U}}\geq 0$ for all t. $\Phi^{\dagger}$ must be substantive (i.e. not zero all the time) when the free market path is declining ($\dot{U}^{\mu}<0$), so it overlaps with the environmental policy as noted in Remark 2.2. To achieve $\dot{\hat{U}}\geq 0$ with minimum loss of PV, the opsustimal policy $\tilde{\Phi}^{\dagger}$ must be non-zero only when utility is constant:

$$\widetilde{\Phi}^{\dagger} = -\dot{\widetilde{\phi}}{}^{\dagger\dagger}_{C}/(1+\widetilde{\phi}{}^{\dagger\dagger}_{C})-\widetilde{\phi}{}^{\dagger\dagger}_{S} = \delta-\dot{A}/A+(\hat{C}\hat{U}_{CS}+\eta\hat{U}_{S})/\hat{U}_{C}A \quad \text{if } \dot{\hat{U}}=0;$$

$$\widetilde{\Phi}^{\dagger} = 0 \text{ otherwise.}$$

However, the implicit nature of the first equation ($\hat{U}$ and $\hat{C}$ are themselves functions of $\Phi^{\dagger}$) makes it hard to deduce the general properties of any SD policy, let alone the opsustimal one. For example, to address one of Krautkraemer's concerns, there is no general result that an SD constraint causes permanent preservation of a positive amount of the resource bounded away from zero. We now consider two special cases of cake-eating economies which illustrate different aspects of the above results.

2.3.2 *Special Case 1: A multiplicative utility function and exponential technical progress*

We assume a multiplicative utility function and exponential technical progress:

$$\left.\begin{array}{llll} & U(C,S) = C^{\nu}S^{\epsilon} & \text{and} & A(t) = e^{\tau t} \\ \text{where} & 0 < \nu,\ \epsilon,\ \nu+\epsilon < 1 & \text{and} & \delta > \tau\nu. \end{array}\right\} \quad (2.19)$$

$U(.)$ here satisfies the marginal conditions (2.4) and (2.5). Since a larger ϵ means that the resource has more amenity value, ϵ will be called the 'environmental concern'. The restriction $\nu+\epsilon<1$ stops it being PV-optimal to consume all the resource stock at time zero. The restriction $\delta>\tau\nu$ ensures that a PV-optimal path exists (if $\delta<\tau\nu$, delaying consumption to the future at any time increases PV). Insertion of (2.19) into (2.9) and manipulating can be shown (see Appendix 2.3) to give the socially PV-optimal consumption path:

$$(1-\nu)\dot{C} = [\tau-\delta+\epsilon(1-\nu)C/\nu S e^{\tau t}]C, \quad (2.20)$$

$$\text{hence} \quad (1-\nu)(\ddot{S}/\dot{S}+\tau) = \tau-\delta-\epsilon(1-\nu)\dot{S}/\nu S. \quad (2.21)$$

The only solution of this which satisfies the initial condition (2.3) is

$$\widetilde{S}(t) = S_0 e^{-\theta t} \quad \text{where} \quad \theta = \nu(\delta - \tau\nu)/[(1-\nu)(\nu+\epsilon)]. \tag{2.22}$$

At first it might seem odd that solving the second-order equation (2.21) did not produce two unknown constants of integration (θ and one other), which were then determined using the terminal (i.e. transversality) condition (2.10) as well as the initial condition. But $C=0$ is technically a solution of (2.20), which accounts for the missing degree of freedom. We can also check that (2.10) does hold, for the growth rate of $\widetilde{U}_C e^{(\tau-\delta)t}\widetilde{S}$ is $[(\nu-1)(\tau-\theta)-(\epsilon+1)\theta+\tau-\delta]t = (\tau\nu-\delta)t/(1-\nu) < 0$ from (2.19). This is sufficient to establish the optimality of the solution path (2.22) (Léonard and Van Long 1992, pp 288-9). From (2.22) and (2.19) the utility path is then

$$\widetilde{U}(t) = \widetilde{U}(0)e^{[\nu(\tau-\theta)-\epsilon\theta]t} = \theta^{\nu} S_0^{\,(\nu+\epsilon)} e^{[\nu(\tau-\delta)/(1-\nu)]t}, \tag{2.23}$$

so the socially PV-optimal path is either sustained (if $\tau \geq \delta$) or always declining (if $\tau < \delta$). An SD policy will thus be needed either never, or always. However, this simple dichotomy is clearly a special feature of this case; as is the fact that sustainedness is unaffected by the environmental concern ϵ, which stems from the coincidence that $U_S = CU_{CS} + \eta U_S$ here.

From (2.18), the free market path has a utility growth rate

$$\dot{U}^{\mu}/U^{\mu} = \nu[\tau - \delta(\nu+\epsilon)/\nu]/(1-\nu-\epsilon) \tag{2.24}$$

and so is sustained if $\tau > \delta(1+\epsilon/\nu)$. In accordance with Remark 2.2, this is a more stringent condition for SD than $\tau > \delta$ for the socially PV-optimal path. There are thus three possible outcomes for the overlap of environmental and SD policies in Special Case 1:

(i) If $\delta \leq \tau/(1+\epsilon/\nu)$, $\dot{U}^{\mu} \geq 0$, i.e. the free market path is sustained, so no SD policy is required per se.

(ii) If $\tau/(1+\epsilon/\nu) < \delta \leq \tau$, then $\dot{U}^{\mu} < 0$ but $\dot{\tilde{U}} \geq 0$, so environmental policy intervention will at the same time make the economy sustained.

(iii) If $\delta > \tau$, both $\dot{U}^{\mu} < 0$ and $\dot{\tilde{U}} < 0$, so environmental policy is not strong enough to achieve SD; the latter requires a stronger policy incentive, which can be justified only by some more egalitarian ethic of intergenerational equity than PV-optimality. Or, if one wished, one could regard τ as the maximum ethically permitted level of the utility discount rate.

Note also: (i) $\partial(\dot{U}^{\mu}/U^{\mu})/\partial\epsilon = -(\delta-\tau\nu)/(1-\nu-\epsilon)^2 < 0$, illustrating Remark 2.1 that a stronger environmental concern makes the free market solution 'less sustained'. (ii) The above results are consistent with the overlapping generations results of Mourmouras (1993, p255) who effectively assumes $\epsilon=0$. Such consistency suggests that not too much is lost by using the representative person framework in this paper.

The actual environmental or SD policies in this case are two constant levels of $\Phi = -\dot{\phi}_C/(1+\phi_C)-\phi_S$. One, the environmental policy $\tilde{\Phi}$, makes the resource depletion rate equal to θ; the other, the SD policy $\Phi^{\dagger}$, makes utility constant. Both can be computed by letting the resource depletion rate in response to a general Φ be an unknown $\psi(\Phi)$, and then noting that

$$S = S_0 e^{-\psi t} \quad \Rightarrow \quad C = \psi S_0 e^{(\tau-\psi)t} \quad \Rightarrow \quad U = \psi^{\nu} {S_0}^{\nu+\epsilon} e^{[\nu(\tau-\psi)-\epsilon\psi]t}. \tag{2.25}$$

The general policy path (2.14) then gives

$$\dot{C}/C = (-\epsilon\psi+\tau-\delta+\Phi)/(1-\nu) = \tau-\psi \quad \text{from (2.25)}$$

$$\Rightarrow \psi(\Phi) = (\delta-\Phi-\tau\nu)/(1-\nu-\epsilon).$$

From (2.22), the environmental policy defined by $\psi(\tilde{\Phi}) = \theta$ is

$$\widetilde{\Phi} = \epsilon(\delta - \tau\nu)/(\nu+\epsilon)(1-\nu) > 0, \tag{2.26}$$

and from (2.25) an appropriate SD policy (assuming $\delta > \tau$ so that one is necessary) is defined by $\dot{U}/U = \tau\nu - (\nu+\epsilon)\psi(\Phi^\dagger) = 0$ always, whence

$$\Phi^\dagger = \delta - \tau\nu/(\nu+\epsilon) > 0 \quad \Rightarrow \quad \psi(\Phi^\dagger) = \tau\nu/(\nu+\epsilon) =: \kappa, \text{ say.} \tag{2.27}$$

This $\Phi^\dagger$ can also be shown (see Appendix 2.4) to be the opsustimal policy $\widetilde{\Phi}^\dagger$ in this case. From (2.26) and (2.27), both environmental and SD policies must be stronger when the utility discount rate δ or the amenity value ϵ increases, or when technical progress τ decreases. And since $\psi(\widetilde{\Phi}^\dagger) < \psi(\widetilde{\Phi})$ if $\delta > \tau$, we then have $\widetilde{C}^\dagger(0) < \widetilde{C}(0)$, so that correcting unsustainedness does require a reduction in initial utility, as anticipated in Section 2.2.

Still assuming $\delta > \tau$, it is possible to compute directly from (2.1) the per capita PV costs of social sustainedness (where everyone acts together to reach $\widetilde{U}^\dagger$) and of individual sustainedness. The latter is the PV that an agent would forgo (compared to the social maximum PV) if she were to achieve sustainedness while all other agents deplete their resource stocks at the socially PV-optimal rate θ. To do this she would deplete her stock at rate ζ such that her utility is constant. From (2.19), this means that $\zeta = \tau - \epsilon\theta/\nu$, which is feasible as long as $\tau > \epsilon\theta/\nu$ ($\Rightarrow \tau > \delta/[1+\nu(1-\nu)/\epsilon]$), which it may well not be. (In saying that "individuals clearly cannot provide for the climate of their offspring acting individually", Howarth and Norgaard 1992 have in mind a case where individual sustainedness is infeasible.) If $\tau > \epsilon\theta/\nu$, the PV of the individually sustained path is strictly less than the PV of the socially (just) sustained path because $\zeta < \kappa = \psi(\Phi^\dagger)$ (see Appendix 2.5); so the cost of individual sustainedness is more than the cost of social sustainedness. This leads to an important general conjecture, which I think almost every proponent of SD policy makes:

CONJECTURE 2.1: *In an economy with cumulative, irreversible externalities, social sustainedness costs less than individual sustainedness (which may be infeasible), so collective action to achieve SD might be justified on standard cost-benefit grounds.*

If each individual values her own sustainedness by an amount commensurate with PV and lying between the per capita costs of social and individual sustainedness, then she will not act alone to achieve sustainedness, because she would lose PV; but a collective sustainedness policy would increase everyone's PV. But if individual sustainedness is feasible and is valued more highly than its cost, individuals will take action to achieve sustainedness anyway, and policy intervention is unnecessary. The same conjecture is made in Chapter 4, where we consider the externalities that arise from intergenerational bequests in a world with different sexes.

2.3.3 Special Case 2: An additive utility function and quadratic technical progress

This case illustrates the implication in (2.11) that a sub-exponential rate of technical progress can result in unsustainedness, and that SD policy intervention to counteract this may not be needed all the time. The utility and technical progress functions are:

$$\left.\begin{array}{llll} & U = \nu\log C + \epsilon\log S,^{17} & \text{and} & A = (1+\tau t)^2 \\ \text{where} & \nu,\ \epsilon > 0 & \text{and} & \tau > 0. \end{array}\right) \quad (2.28)$$

17. Although $U=\nu\log C+\epsilon\log S$ is only a monotonic transformation of $U=C^{\nu}S^{\epsilon}$, the utility function in Special Case 1, it does make a difference since dynamic utility maximisation assumes the intertemporal comparability of utility at each time.

Substituting this into (2.9) and using the initial condition $\widetilde{S}(0) = S_0$ can be shown (see Appendix 2.6) to give the unique socially PV-optimal path

$$\left.\begin{aligned} \widetilde{S} &= S_0 e^{-\delta\nu t/(\nu+\epsilon)} \\ \widetilde{U} &= 2\nu\log(1+\tau t) - \delta\nu t + \text{constant} \end{aligned}\right) \quad (2.29)$$

The same remarks about not needing to use the transversality condition to find the unique solution apply here as in Section 2.3.2; and the transversality condition does hold, since $\widetilde{U}_C A e^{-\delta t}\widetilde{S} = e^{-\delta t}(\nu+\epsilon)/\delta \to 0$ as $t \to \infty$. Note from (2.29) that the socially PV-optimal path is always

unsustained, since $\dot{\widetilde{U}} = [2\tau/(1+\tau t)-\delta]\nu = (\dot{A}/A-\delta)\nu$, which eventually becomes negative even if $2\tau>\delta$ so that utility is initially rising. The root cause of unsustainedness here is clearly the failure of the technical progress rate $\dot{A}/A$, the incentive to conserve the resource, to keep ahead of the constant utility discount rate δ, the incentive to consume it now.

From (2.14), if $\Phi=-\dot{\phi}_C/(1+\phi_C)-\phi_S$ is constant, the policy path can be shown (see Appendix 2.6) to be:

$$\left.\begin{aligned} \hat{S} &= S_0 e^{-(\delta-\Phi)t} \\ \hat{U} &= 2\nu\log(1+\tau t) - (\nu+\epsilon)(\delta-\Phi)t + \text{constant} \end{aligned}\right) \quad (2.30)$$

On the free market path (i.e. with policy strength $\Phi=0$), utility growth $\dot{U}^\mu=[2\tau/(1+\tau t)-\delta]\nu-\delta\epsilon$ falls over time, again giving a decay from a rising path (if $2\tau>\delta(1+\epsilon/\nu)$) to an eventually declining one. The extra $-\delta\epsilon$ term makes utility growth lower on the free market than on the socially optimal path, in accordance with Remark 2.2. Comparing (2.29) and (2.30) shows that environmental policy in this model is $\widetilde{\Phi}=\delta/(1+\nu/\epsilon)$, which has to become stronger as either the utility discount rate δ or environmental concern ϵ rises.

As for sustained paths, a maximum, constant, positive utility path does exist and can be shown (see Appendix 2.7), by directly differentiating (2.28), to be:

$$S_0^m = S_0(1+\tau t)^{-\nu/(\nu+\epsilon)} \quad \text{(no longer an exponential decline)} \tag{2.31}$$

$$U_0^m = (\nu+\epsilon)\log S_0 + \nu\log[\nu/(\nu+\epsilon)],$$

There is no simple expression for a general constant utility policy here, since for $\hat{S}$ in (2.30) to have the same time dependence as S_0^m, Φ could not be constant, but that would invalidate the derivation of $\hat{S}$. If $2\tau > \delta(1+\epsilon/\nu)$ so that the free market path is initially rising, the PV-optimal SD (opsustimal) policy will have two distinct phases, attaining constant utility only after some finite period of rising utility. If $2\tau \leq \delta(1+\epsilon/\nu)$, the opsustimal path will be (2.31), and one might hope to be able to compute the cost of sustainedness $\int_0^\infty (\tilde{U} - U_0^m)e^{-\delta t}dt$. Unfortunately we cannot, because $\int_0^\infty [2\nu\log(1+\tau t) - \delta\nu t + \nu\log\delta]e^{-\delta t}dt$ has a $\log(1+\tau t)$ term in it.

2.4 CAPITAL-RESOURCE SUBSTITUTION

In a cake-eating economy, there is no difference between choosing to consume rather than save and invest, and choosing to deplete the resource rather than conserve it. In a productive economy, where accumulated capital or technical progress can be substituted for the resource as an input to production, these two choices are distinct. Consumption taxes and resource taxes therefore have different roles as SD policies, with the potential of the latter being severely limited by the finiteness of the non-renewable resource, as we shall see. Section 2.4.1 analyses SD and environmental policies in a general economy with a resource amenity, capital-resource substitution and exogenous Hicks-neutral technical progress (we model no other type of technical progress here). Section 2.4.2 further assumes Cobb-Douglas utility and production functions, and derives an exact solution for the steady state. In all the models in this section, there is no explicit capital market, so that saving is automatically assumed to be invested.

2.4.1 The general model

The model is as set out in Section 2.3.1, except that a capital stock $K(t)$ and production function $F(K,R)$ now enter the relationship between consumption C and resource depletion R:

$$C(t) = A(t)F(K,R) - \dot{K} \quad \text{where} \tag{2.32}$$

$$K(0) = K_0 > 0 \text{ and } K \geq 0 \text{ for all } t.$$

The production function $F(K,R)$ is assumed to have diminishing returns to each of its separate inputs, and to be twice continuously differentiable and linearly homogeneous.[18] It can therefore be written in the intensive form $F(K,R)=Rf(x)$, where $x=K/R$ is the capital/resource input ratio. In addition we make the crucial, pessimistic assumption that the marginal productivity of capital is ultimately less than the utility discount rate:

$$\lim_{x\to\infty} F_K = \lim_{x\to\infty} f'(x) < \delta \tag{2.33}$$

As we shall see, this may cause the PV-optimal paths to be unsustained, depending on the technical progress rate $\dot{A}/A$. The resource is assumed to be 'optimally essential' in the sense of Dasgupta and Heal (1974, p15,

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18. As discussed in Chapter 3, assuming constant returns in just K and R, as opposed to in K, R and labour L, is mathematically important but physically questionable. The special case below shows that assuming diminishing returns instead may result in environmental policy having a different effect on the interest rate.

footnote), in that either $F(K,0)=0$ or $\lim_{R\to 0}F_R=\infty$, so that $R>0$ always on a PV-optimal path.[19] We adopt the same ad hoc technique as in Section 2.3 and do not attempt to include the SD constraint (2.6) in the formal optimisation. The equivalents of Krautkraemer's equations (2.20) and (2.21) for an interior solution of the socially PV-optimal path can be shown (see Appendix 2.8) to be (2.34), effectively the Ramsey rule for socially PV-optimal saving, which with (2.6) gives the utility path (2.35); and (2.36), effectively the Hotelling rule for Pareto-optimal resource depletion:

$$\dot{\widetilde{C}}/\widetilde{C} = (A\widetilde{f}\,' - \delta - \widetilde{R}\widetilde{U}_{CS}/\widetilde{U}_C)/\eta \quad \text{where } \widetilde{f}\,' := f'(\widetilde{x}), \text{ etc;} \tag{2.34}$$

hence $$\dot{\widetilde{U}} = [(A\widetilde{f}\,' - \delta)\widetilde{U}_C\widetilde{C} - \widetilde{R}(\widetilde{C}\widetilde{U}_{CS} + \eta\widetilde{U}_S)]/\eta \tag{2.35}$$

$$\dot{\widetilde{x}}/\widetilde{x} = (A - \dot{A}/A\widetilde{f}\,')\sigma(\widetilde{x})\widetilde{f}/\widetilde{x} \;-\; \widetilde{U}_S/\widetilde{U}_C A\widetilde{x}^2(-\widetilde{f}\,'') \qquad (2.36)^{20}$$

where σ is the elasticity of substitution between capital and resource inputs. The socially PV-optimal path is then determined by equations (2.34) and (2.36), the transversality condition, and the initial stocks of capital and resource. Note that there is no U_S term in (2.34). Concern for conserving the social resource amenity (as opposed to the 'live now, pay later' U_{CS} effect of declining resource amenity on consumption) enters through the Hotelling ($\dot{x}/x$) equation for optimal resource depletion, and then only in the

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19. The essentiality of consumption follows from (2.5). Krautkraemer implicitly assumes resource essentiality when he looks (p159) for an interior solution.

20. An intermediate version of this is that $\dot{F}_R = F_K F_R - U_S/U_C$ on an efficient resource depletion path. Using this it can be shown (see Appendix 2.9) that if there is no technical progress and the amenity value of the resource is internalised into the competitive resource price F_R, the form of Hartwick's rule is unchanged in this economy, i.e investing resource rents ($\dot{K}=RF_R$) results in constant utility forever. Intuitively, if the resource is correctly priced, $\dot{K}=RF_R$ means zero aggregate investment, which maintains constant aggregate wealth and hence constant utility in the absence of technical progress.

socially PV-optimal case. This also means that the U_S (as opposed to ηU_S) term that was in the cake-eating utility equation (2.11) is no longer present. In the capital-resource model, technical progress ($\dot{A}>0$) therefore plays an indispensable role in making the socially PV-optimal path sustained:

PROPOSITION 2.1: *In the capital-resource substitution model with no technical progress, the socially PV-optimal path is unsustained.*

PROOF:
From Krautkraemer's Lemma A3, $\lim_{t\to\infty} x=\infty$ on the socially PV-optimal path. So if productivity A is constant ($=1$), from (2.33) the first term and hence the whole right hand side of (2.35) is asymptotically negative, resulting in permanently declining utility after a finite time. (This is untrue if there is technical progress, since then $A\tilde{f}'$ may stay greater than δ.) ||

Policy instruments considered are specific taxes $\phi_C(t)$ on consumption and $\phi_S(t)$ on the resource stock as before, and now also taxes $\phi_R(t)$ on resource depletion and $\phi_K(t)$ on the capital stock. It can be shown (see Appendix 2.10) that these give the following equations for the paths of consumption, utility and the capital/resource ratio:

$$\dot{\hat{C}}/\hat{C} = (A\hat{f}' - \delta + \Phi_C - \phi_K - \hat{R}\hat{U}_{CS}/\hat{U}_C)/\eta \tag{2.37}$$

$$\dot{\hat{U}} = [(A\hat{f}' - \delta + \Phi_C - \phi_K)\hat{U}_C\hat{C} - \hat{R}(\hat{C}\hat{U}_{CS} + \eta\hat{U}_S)]/\eta \tag{2.38}$$

$$\dot{\hat{x}}/\hat{x} = [A - (\dot{A}/A + \phi_K)/\hat{f}']\hat{\sigma}\hat{f}/\hat{x} - \Phi_x/\hat{x}^2(-\hat{f}'') \tag{2.39}$$

where $\hat{f}' = f'(\hat{x})$, etc,

$$\Phi_C := -\dot{\phi}_C/(1+\phi_C) \quad \text{and} \quad \Phi_x := \hat{f}'\phi_R - (\dot{\phi}_R + \phi_S + \phi_K\phi_R)/A.$$

Similarly to the socially PV-optimal case, (2.37), (2.39), the transversality condition and the initial stocks then determine the path.

Comparison of (2.34) and (2.36) with (2.37) and (2.39) respectively shows that an *environmental* policy will always be needed, and

$$\widetilde{\Phi}_C - \widetilde{\phi}_K = 0 \quad \text{and} \quad \widetilde{\Phi}_x = \widetilde{U}_S / \widetilde{U}_C A > 0$$

form the required policy. While a non-zero capital tax $\widetilde{\phi}_K$ is technically consistent with this, it is hardly worth introducing it only to cancel it with a non-zero $\widetilde{\Phi}_C$. So in the capital-resource substitution model, regardless of whether or not there is technical progress, either a positive but decreasing resource depletion tax, or a resource stock subsidy, or a combination of the two, form the appropriate environmental policy; while consumption and capital taxes play no part in it (i.e., $\widetilde{\Phi}_C = \widetilde{\phi}_K = 0$).

The impact effect of $\Phi_x > 0$ is to make x rise more slowly. Since $f''(x) < 0$, this suggests that environmental policy raises the interest rate $f'(x)$ and (from (2.38)) also the rate of change of utility. However, this is not guaranteed because other terms in (2.37) and (2.39) will also be affected by the policy Φ_x. And we see below that environmental policy can lower the interest rate if there are diminishing rather than constant returns to K and R.

As for *SD* policies, SD will not be achieved without adequate levels of saving and investment. This has long been recognised in the theoretical literature on Hartwick's rule (and recently stressed by Solow 1993), and recent empirical work such as Pearce and Atkinson (1993) shows how variations in savings rates are important in explaining international variations in sustainedness. But neither type of literature has studied how policies can achieve the required levels of saving in market economies; Solow (p171) merely says that the "concrete translation of sustainability into policy leaves a lot of questions unanswered." Some of these questions can be addressed by the above models, where there is a sharp contrast between the power of resource taxes and the power of a consumption or capital tax.

The effect of resource taxes on sustainedness is indirect and ambiguous, since the taxes act through both the interest rate $f'(K/R)$ and the resource flow R and stock S. At any given time, to encourage more saving, the interest rate needs to be raised, which means (for any given capital stock) increasing the resource flow. But to reduce the harmful effect on utility growth of the amenity-related term $-R(CU_{CS}+\eta U_S)$, resource flow needs to be decreased and the stock increased. This gives the intuition behind:

PROPOSITION 2.2: *In the capital-resource substitution model with no technical progress, resource taxes cannot achieve SD.*

PROOF:
By (2.38), if the technical productivity factor A is constant, the only way that resource taxes ϕ_R or ϕ_S might achieve SD is by keeping x bounded such that $Af'(x) > \delta$. But since the resource is finite and non-renewable, $\lim_{t\to\infty}R=0$. Bounded x $(=K/R)$ then means that $\lim_{t\to\infty}K = \lim_{t\to\infty}F = \lim_{t\to\infty}C = 0$. But $\lim_{t\to\infty}C = 0$ contradicts the deduction from (2.6) that consumption must rise on a sustained path. ||

This insufficiency of resource taxes to achieve SD is a noteworthy result (although, because they can achieve social PV-optimality, such taxes are probably necessary to achieve SD at minimum cost). In contrast, because a consumption or capital tax acts directly on the consumption-saving decision, even if A is constant, it is always theoretically possible to find such taxes such that $\dot{\hat{U}}=0$, namely:

$$-\dot{\phi}_C^{\dagger}/(1+\phi_C^{\dagger})-\phi_K^{\dagger} = \hat{R}(\hat{C}\hat{U}_{CS}+\eta\hat{U}_S)/\hat{U}_C\hat{C} + \delta - A\hat{f}'$$

which from (2.38) will achieve constant utility. But, with constant A and hence $\lim_{t\to\infty}A\hat{f}'<\delta$, $-\dot{\phi}_C^{\dagger}/(1+\phi_C^{\dagger})-\phi_K^{\dagger}$ must be asymptotically positive, so either $\phi_C^\dagger$ must again (by Lemma 2.1) approach a 100% subsidy, or ϕ_K must become a capital subsidy. Either subsidy would in practice cause the political problems of lump sum taxes, as previously noted. So in the absence of technical progress, practical SD policies are still hard to find, even given unlimited substitution of capital for natural resources. Intuitively, achieving constant utility is bound to be difficult in the long term, once capital has been piled up and resources eaten away to the point where PV-maximising individuals want to consume capital, rather than accumulate more of it for an inadequate return.

No exact solutions for the socially PV-optimal or maximum constant utility paths appear to be possible. Attempting a solution with both production and utility functions being Cobb-Douglas ($F = K^\alpha R^{1-\alpha}$ and $U = C^\nu S^\epsilon$) leads to insoluble differential equations. However, one can at least establish the *feasibility* of sustained utility in this case when $\alpha > 0.5$, following Solow (1974). Consider the path:

$$K = K_0 + ht,\ h > 0;\quad R = b(K_0+ht)^{-\alpha(1-z)/(1-\alpha)};\quad 0 < z < 2 - 1/\alpha,\ b > 0$$

$$\Rightarrow \qquad C = F(K,R) - \dot{K} = K^\alpha R^{1-\alpha} - h = b^{1-\alpha}(K_0+ht)^{\alpha z} - h.$$

Total resource depletion $\int_0^\infty R dt$ is bounded on this path, and less than the initial stock S_0 if the constant b is small enough, so $\lim_{t\to\infty} S > 0$. Provided the constant h is also small enough, $C(0) > 0$ so that consumption is positive, rising and unbounded, and hence $U = C^\nu S^\epsilon$ is eventually rising and unbounded, which means a sustained path must be feasible, by the 'ironing-out' argument set out in Section 2.2.

2.4.2 *Steady-state solutions in a Cobb-Douglas case with technical progress*

Following Stiglitz (1974), we can find exact expressions for the PV-optimal, asymptotic, steady-state paths in our capital-resource economy when there is exponential technical progress and both production and utility functions are Cobb-Douglas:

$$\left.\begin{aligned} Q(K,R) &= e^{\tau t}K^{\alpha}R^{1-\alpha}, \quad 0<\alpha<1 \\ U(C,S) &= C^{\nu}S^{\epsilon}, \qquad 0 < \nu,\ \epsilon,\ \nu+\epsilon < 1 \\ \delta &> \tau\nu/(1-\alpha) \ \text{ to ensure that PV converges.} \end{aligned}\right) \qquad (2.40)$$

It can be shown (see Appendix 2.11) that the socially PV-optimal growth rates are then:

$$\left.\begin{aligned} (1-\nu)\dot{\widetilde{C}}/\widetilde{C} &= \alpha e^{\tau t}/\widetilde{x}^{1-\alpha} - \delta - \epsilon\widetilde{R}/\widetilde{S} \\ \dot{\widetilde{x}}/\widetilde{x} &= (e^{\tau t}-\tau\widetilde{x}^{1-\alpha}/\alpha)\widetilde{x}^{\alpha-1} - \epsilon\widetilde{C}/\nu\widetilde{S}e^{\tau t}\alpha(1-\alpha)\widetilde{x}^{\alpha} \\ \dot{\widetilde{Q}}/\widetilde{Q} &= \alpha\dot{\widetilde{K}}/\widetilde{K} + (1-\alpha)\dot{\widetilde{R}}/\widetilde{R} + \tau. \end{aligned}\right) \qquad (2.41)$$

Now define

$$\partial Q/\partial K =: r(t); \ \dot{K}/K =: \xi(t), \quad Q/K =: r(t)/\alpha, \quad R/S =: \gamma(t)$$

Since the economy is assumed to be competitive, r, the marginal product of capital, is also the interest rate. In the asymptotic socially PV-optimal steady state, $\dot{\widetilde{C}}/\widetilde{C}$, $\dot{\widetilde{K}}/\widetilde{K}$ and $\dot{\widetilde{Q}}/\widetilde{Q}$ all tend to the same constant, say $\widetilde{\xi}^*$, while $\widetilde{Q}/\widetilde{K}$ tends to some constant $\widetilde{r}^*/\alpha$ and $\dot{\widetilde{R}}/\widetilde{R}$ tends to some constant $-\widetilde{\gamma}^*$. It can then be shown (see Appendix 2.12) that

$$\tilde{\gamma}^* = [\delta - \tau\nu/(1-\alpha)]/(1-\nu-\epsilon) \;-\; \epsilon(\tilde{r}^*/\alpha - \tilde{\xi}^*)\tilde{\gamma}^*\alpha/\nu(1-\alpha)(1-\nu-\epsilon)\tilde{r}^* \tag{2.42}$$

$$\tilde{r}^* = (1-\nu)\tau/(1-\alpha) + \delta - (1-\nu-\epsilon)\tilde{\gamma}^* \tag{2.43}$$

$$\tilde{\xi}^* = \tau/(1-\alpha) - \tilde{\gamma}^* \tag{2.44}$$

$$\dot{\tilde{U}}^*/\tilde{U}^* = \nu\tilde{\xi}^* - \epsilon\tilde{\gamma}^* = \tau\nu/(1-\alpha) - (\nu+\epsilon)\tilde{\gamma}^*. \tag{2.45}$$

Trying to eliminate $\tilde{r}^*$ and $\tilde{\xi}^*$ from the first three equations gives a complicated quadratic in $\tilde{\gamma}^*$. Rather than solving this explicitly, note that on the free market path, the fact that agents ignore the effect of their resource depletion on amenity eliminates the second term on the right hand side of (2.42). This makes the equations linear, with solution:

$$\left.\begin{aligned}
\gamma^{\mu*} &= [\delta - \tau\nu/(1-\alpha)]/(1-\nu-\epsilon) \\
r^{\mu*} &= \tau/(1-\alpha) \\
\xi^{\mu*} &= [\tau(1-\epsilon) - \delta(1-\alpha)]/(1-\alpha)(1-\nu-\epsilon) \\
\dot{U}^{\mu*}/U^{\mu*} &= [\tau\nu - \delta(\nu+\epsilon)(1-\alpha)]/(1-\alpha)(1-\nu-\epsilon).
\end{aligned}\right) \tag{2.46}$$

So asymptotically,[21] the free market resource depletion rate $\gamma^{\mu*}$ rises and sustainedness $\dot{U}^{\mu*}/U^{\mu*}$ worsens as either technical progress falls, or the discount rate rises, or environmental concern strengthens (since $\partial[\dot{U}^{*\mu}/U^{*\mu}]/\partial\epsilon < 0$). For the free market path to be sustained requires a minimum level of exogenous technical progress ($\tau > \delta(1+\epsilon/\nu)(1-\alpha)$); or as with the first cake-eating special case, allows a maximum, ethically acceptable utility discount rate, if one prefers to view it that way. The fact that the interest rate $r^{\mu*}$ is independent of both the discount rate and

21. To save repetition, the 'asymptotic' or 'steady-state' qualifier applies to the appropriate variables throughout the rest of this section, unless otherwise stated.

environmental concern is a quirk of this special case, as will be shown later.

Since we require $Q > \dot{K}$ and therefore $\tilde{r}^*/\alpha > \tilde{\xi}^*$ for a meaningful solution, the ϵ term in (2.42), which represents the effect of environmental policy, is negative. Therefore, unsurprisingly, environmental policy slows resource depletion ($\tilde{\gamma}^* < \gamma^{\mu*}$). In turn, this means from (2.43), (2.44) and (2.45) respectively that environmental policy raises the interest rate ($\tilde{r}^* > r^{\mu*}$), the growth rate of production, consumption and investment ($\tilde{\xi}^* > \xi^{\mu*}$), and lastly sustainedness ($\dot{\tilde{U}}^*/\tilde{U}^* > \dot{U}^{\mu*}/U^{\mu*}$).

As for environmental or SD policies, from (2.37)-(2.39) the privately PV-optimal path in response to tax policies $\Phi_C = -\dot{\phi}_C/(1+\phi_C)$, ϕ_K and $\Phi_x = \hat{f}'\phi_R - (\dot{\phi}_R + \phi_S + \phi_K\phi_R)/e^{\tau t}$ can be shown (see Appendix 2.13) to have the following rates of resource depletion, interest, consumption growth and utility growth (which $\Phi_C = \phi_K = \Phi_x = 0$ reduces to the free market equations (2.46)):

$$
\left.
\begin{aligned}
\hat{\gamma}^* &= [\delta - \tau\nu/(1-\alpha) - \Phi_C - \Phi_x/(1-\alpha)\hat{x}^\alpha]/(1-\nu-\epsilon) \\
\hat{r}^* &= \tau/(1-\alpha) + \phi_K + \Phi_x/(1-\alpha)\hat{x}^\alpha \\
\hat{\xi}^* &= \{\tau(1-\epsilon)/(1-\alpha) - [\delta - \Phi_C - \Phi_x/(1-\alpha)\hat{x}^\alpha]\}/(1-\nu-\epsilon) \\
\dot{\hat{U}}^*/\hat{U}^* &= \{\tau\nu/(1-\alpha) - (\nu+\epsilon)[\delta - \Phi_C - \Phi_x/(1-\alpha)\hat{x}^\alpha]\}/(1-\nu-\epsilon)
\end{aligned}
\right) \qquad (2.47)
$$

No expression for the required environmental policy is given here since (2.42)-(2.45) were not solved explicitly. From $\dot{\hat{U}}^*/\hat{U}^*$, if $\tau\nu < \delta(1-\alpha)(\nu+\epsilon)$ the SD policy is formally

$$\Phi_C^\dagger + \Phi_x^\dagger/(1-\alpha)\hat{x}^\alpha = \delta - \tau\nu/(1-\alpha)(\nu+\epsilon)$$

which is similar to (2.27) in the cake-eating case. Policy must be stronger as the discount rate or environmental concern rises, or as the technical progress rate falls. But the consumption tax ϕ_C again has to tend to a 100%

subsidy to raise utility growth; and the asymptotically zero multiplying factors f', $1/e^{\tau t}$ and $1/x^{\alpha}$ in $\Phi_x/(1-\alpha)\hat{x}^{\alpha}$ mean that a capital subsidy, resource depletion tax and/or resource stock subsidy would have to grow without bound to be effective. So even when technical progress $\tau > 0$, it is still remarkably difficult to find effective instruments of SD policy.

The model parameters and policies affect the asymptotic interest rate in quite a different way to $\hat{r}^*$ in (2.47) if one assumes diminishing rather than constant returns. If the production function includes a third, constant input such as a constant labour force L:

$$Q(K,R) = e^{\tau t}K^{\alpha}R^{\pi}L^{1-\alpha-\pi} \quad 0 < \alpha,\ \pi,\ \alpha+\pi < 1; \tag{2.48}$$

it can be shown (see Appendix 2.14) that the asymptotic interest rate on the privately PV-optimal path with a consumption policy of strength Φ_C, a capital subsidy of $(-\phi_K)$ and a conservation policy of strength Φ_Σ, which improve sustainedness (though it is not possible to compute the correct level of Φ_Σ required for social PV-optimality), is:

$$r = \{(\delta-\Phi_C+\phi_K)(1-\alpha-\pi) + \tau(1-\epsilon-\nu) + \Phi_\Sigma[\pi(1-\nu)-\epsilon(1-\alpha)]\} \\ / \ [(1-\epsilon)(1-\alpha)-\pi\nu] \tag{2.49}$$

Both the discount rate δ and the environmental concern ϵ affect the interest rate here. Using a consumption tax or capital subsidy as an environmental and/or SD policy (i.e. choosing $\Phi_C - \phi_K > 0$) will lower the interest rate, whereas using a conservation policy ($\Phi_\Sigma > 0$) will lower *or raise* the interest rate, according to whether $\pi(1-\nu)$ is less or greater than $\epsilon(1-\alpha)$. This lack of simple connection between the interest rate and environmental or SD policies should not be surprising since, at least in theoretical growth models, the interest rate is not a policy tool, but an endogenously-determined price which balances the supply and demand of funds for capital investment.

An asymptotic steady state will be of little interest to real policymakers if it takes decades before the economy is close to it. The question of what happens on the (PV-optimal) *approach* to a steady state is therefore very important. Unfortunately, even if a particular steady state (whether privately or socially PV-optimal) is sustained, this is no guarantee that the approach to that state will also be sustained (see Appendix 2.15 for detailed discussion); but there is little more that can be said apart from this. We also cannot say anything about the cost of sustainedness, since equations (2.41) for the approach to the steady state have no exact solution. Even if parameter values are chosen specially so that the socially PV-optimal solution starts at the steady-state equilibrium (and is unsustained), the constant utility solution would then be starting away from its steady-state equilibrium, which requires different parameter values.

2.5 INITIAL CONDITION EFFECTS

In defining the PV-optimal or opsustimal path of development, there has so far been no constraint (other than non-negativity) on the initial level of consumption, $C(0)$ (which also determines the initial level of utility, $U(0)=U[C(0),S_0])$. But the real world does have a history, which in practice might well constrain the initial levels of consumption and utility to be C_G and $U_G=U(C_G,S_0)$, say. There might also be unexpected changes in the initial resource stock S_0 (e.g. through new discoveries). What effects might these extra or altered initial conditions have on our analysis? We can distinguish at least four important questions of interest. Unfortunately these are all analytically quite difficult questions, and I give only some preliminary thoughts on how to answer them.

(i) If U_G is sustainable but above the opsustimal starting level of utility, what is the PV-optimal sustained path starting from U_G?

This is a well-defined question, but given our earlier difficulty in finding the general opsustimal path when there is no constraint on initial consumption, the answer will not be any easier when there is a constraint. Suppose, for example, that the (unconstrained) PV-optimal path has growing utility forever, and is therefore also the opsustimal path. It is not obvious whether having to start from a higher than opsustimal initial utility level results in a constrained opsustimal path which first has constant utility, and then grows at the unconstrained opsustimal rate; or instead has a lower growth rate throughout. In the exponential steady-state cake-eating case of Section 2.3.2, it is not even possible to calculate analytically a constant utility path starting from a general C_G, since the resource stock $S(t)$ generating this path would need to satisfy both the non-linear differential equation $\nu(\ddot{S}/\dot{S}-\tau)+\epsilon\dot{S}/S=0$ and the initial conditions $S(0)=S_0$, $\dot{S}(0)=-C_G$.

(ii) If U_G is unsustainable, what is the best development path starting from U_G?

This is not a well-defined question without some measure, like an adjustment cost formula, of the effect of declining utility (which is inevitable at some stage, since U_G is unsustainable) on intertemporal social welfare. Without this we do not know what is meant by the 'best' development path. The analysis of maximising social welfare which includes dynamic adjustment costs is left as a challenging and important subject for further research.

(iii) If U_G is sustainable, is it possible for a rise in the initial resource stock S_0 (e.g. through new resource discoveries) to *harm* sustainability? That is, could the initial utility level $U(C_G,S_0)$ ever rise faster than the maximum sustainable utility level $U_0^m(C_G,S_0)$ as S_0 rises, and thus move from a sustainable to an unsustainable value?

The answer is Yes in the case of *renewable* resources where there is a finite carrying capacity $\overline{S}$ and where the harvest is consumed directly (so $R=C$). **Figure 2.3** plots resource harvest against resource stock in such a case, with OADE$\overline{S}$ as the bell-shaped resource growth function. An example would be equation (1.1) where $\overline{S}$ is the carrying capacity and $\dot{S}=\gamma S(\overline{S}-S)-C$ is the equation of motion of the resource stock. The curves convex to the origin are the indifference curves of the utility function $U(C,S)$. Point A with consumption C^* and stock S^* attains the maximum sustainable utility level. An exogenous increase in initial stock S_0 from S^* at A to S_G at B, while retaining the given initial consumption level C^*, would therefore push the initial utility to the unsustainable level $U(C^*,S_G)$. But a higher initial stock would not always harm sustainability. For example, if the given initial state of the economy was originally at E for some reason, an increase in initial resource stock represented by a move to F would neither harm nor help sustainability (though it would increase PV). The economy could move along the indifference curve FD by gradually increasing the harvest. It could thus eventually reach the maximum sustainable utility point A, which it could do anyway from E.

However, the effect of a higher S_0 can be very different with the non-renewable or exponentially expanding resources which are the subject of this thesis. For example, with the cake-eating special case of Section 2.3.2, the maximum sustainable utility level is proportional to $S_0^{\epsilon+\nu}$, and thus increases faster with S_0 than initial utility would if consumption is historically given as C_G, since then $U(0)$ is proportional to S_0^{ν}. However, obtaining a general proof that higher S_0 helps sustainability in the non-renewable resource case would be very hard, because we know so little in general about the maximum sustainable utility level as a function of the initial resource stock; for example, we do not know it in the special Cobb-Douglas case of Section 2.4.2. But since the notion of carrying capacity does not exist for non-renewable or expanding resources, it is hard to see intuitively how an increase in initial resource stock could harm sustainability. One might also argue that, in practice, sudden discoveries of resource stocks are less likely if the resource in question is on the earth's surface and gives amenity value via the utility function $U(C,S)$, rather than if it is say an underground mineral which is of uncertain extent but is unlikely to have amenity value.

(iv) Could the present value of the opsustimal path, $\mathrm{PV}[\widetilde{U}^{\dagger}(K_0,S_0)]$, ever be decreasing in the initial resource stock S_0?

The answer here is obviously Yes, even with non-renewable resources. This is simply because having a given initial consumption level different from the initial opsustimal level is an extra constraint which cannot possibly increase the PV of the opsustimal path, and will generally reduce it. The latter is certainly true for the cake-eating case of Section 2.3.2, where if $\tau > \delta$ the PV-optimal path has rising utility and is unique, so that any feasible path constrained to start from a different level of utility will have lower PV.

2.6 CONCLUSIONS

The widespread modern acceptance of sustainable development (SD) as a policy goal logically entails three beliefs (after Pearce et al 1993, p11): (i) SD is a form of intergenerational equity which should be achieved if it can be; (ii) it may well not be achieved if current development paths are followed; but (iii) policy intervention can achieve it. This chapter has shown that defining SD as forever non-declining utility (NDU), and treating it as a constraint on rather than some kind of alternative to PV-optimality produces no self-evidently absurd results, and several useful ones such as avoiding the well-known poverty trap of the maximin criterion. But we have not tried to justify belief (i) philosophically, or beliefs (ii) and (iii) empirically. The emphasis has been on using neoclassical theory to reach positive conclusions about what causes declining utility during development, and what policies can prevent it.

To do this we analysed two perfect-foresight, neoclassical growth models, in both of which representative agents deplete their private stocks of a finite, non-renewable, essential resource, whilst deriving amenity value from the total resource stock. In the first, cake-eating, model the resource was directly consumed, and in the second, capital-resource, model it was an input to production. The models gave some useful general insights into SD defined as NDU, albeit ones which are often hard to prove precisely.

Two such insights were that a higher resource amenity value (i.e. a stronger private concern for the environment) makes utility 'more declining' and so 'worsens sustainedness' on the free market path of the economy which ignores the social amenity cost of resource depletion; and that the environmental policy which internalises this cost makes utility 'less declining' and so 'improves sustainedness' in the models. In general, though, we do not know when, or by how much sustainedness will improve, or when unsustainedness (actual declines in utility) will happen or will be avoided. (We also do not know whether the interest rate will rise or fall, highlighting the fact that interest rates are not policy instruments in equilibrium growth models.) Contrasting special cases showed a range of sustainedness behaviours: sometimes the free market path has falling utility always, sometimes it has rising utility always, and sometimes utility at first rises, but must eventually fall. Much the same range of behaviour is observed when environmental policy is applied. But this range does at least show that SD policy, contrary to the way many policymakers talk about it, is conceptually distinct from, and may have to go further than, environmental policy.

Technical progress (here assumed to be exogenous) plays a crucial role in SD. It provides the sole incentive to save rather than consume in the cake-eating model. Assuming that the marginal productivity of capital eventually declines below the utility discount rate, environmental policy cannot achieve SD in the capital-resource model if there is no technical progress. And variations in the level and time path of technical progress explain the range of free market sustainedness behaviours noted above in the special cases.

In the simplest cake-eating special case, SD policy can be justified on conventional cost-benefit grounds because individuals care about their own future being sustained, but cannot achieve this efficiently, or perhaps at all, through individual action. An important, unproved conjecture is that a similar justification exists in more general economies, wherever externalities arise from cumulative depletion of non-renewable resources.

The analysis of SD policies in the capital-resource model shows that a resource stock or flow tax plays quite a different role from a consumption or capital tax (in contrast to the less realistic cake-eating case where all the taxes play an equivalent role). Even though resource taxes improve sustainedness, they may be powerless, and are certainly less effective than a consumption or a capital tax, in achieving a sustained path. Secondly, and discouragingly, SD policies in all models eventually become *subsidies* (sometimes 100% subsidies asymptotically). The lump sum taxes then needed to restore revenue neutrality would in practice (to depart from the standard but unrealistic assumption of identical agents) be politically unpopular for reasons of int*ra*generational inequity. One policy not modelled here, reducing taxes on investment income, also involves a loss in revenue, and in practice would also have an adverse effect on intragenerational equity. Neither of these results should be surprising, since attaining constant utility is inevitably difficult in the long term, if the return on investment is such that PV-maximising individuals wish to consume rather than save once capital has been accumulated and resources depleted beyond some point.

A natural direction for further work in this area would be to explore the many alternative assumptions that could have been made in the growth models above, and which may generate fairly different conclusions about SD and environmental polices to those reached here. For example, one could assume uncertainty rather than perfect foresight; endogenous rather than exogenous technical change; inequalities in income and wealth rather than identical agents; and different types of environmental features, such as a renewable rather than a non-renewable resource, a flow rather than a stock externality, or cumulative pollution rather than resource depletion. Criteria of intergenerational welfare which trade off sustainedness with present value maximisation using some finite weighting scheme should also be explored, since they may be politically more credible than using a rigid NDU constraint regardless of its PV cost. But to the extent that the finiteness of natural resources reduces the return to capital investment over time, finding acceptable *incentives to save* which are not pure subsidies, and are not overly harmful to intragenerational equity, seems likely to remain a central problem for SD policy – a problem which should therefore be as much a part of the debate on SD as concerns about global warming or tropical rainforests.

APPENDICES TO CHAPTER 2

All superscripts for different paths ($\tilde{}$, $\hat{}$, $^{\mu}$, †, *) are dropped here for brevity. Equation numbers from the main paper are used where appropriate. Derivations of $\dot{U}$ from $\dot{C}$ using $\dot{U} = U_C\dot{C} - RU_S$ are simple and therefore omitted. A useful general result from (2.2) is

$$\dot{U}_C = U_{CC}\dot{C} + U_{CS}(-R) = -\eta U_C\dot{C}/C - CU_{CS}/A \qquad \text{(A2.1)}$$

We ignore the degenerate solution $C=0$ of many of the differential equations, which is why we end up needing only one boundary condition to give a fully-determined solution to a second-order equation.

Appendix 2.1 General cake-eating model: Derivation of socially PV-optimal path (2.9)

When $\lambda_U=0$, (2.7) and (2.8) respectively become

$$U_C - \pi_S/A = 0 \qquad \text{(A2.2)}$$

$$U_S = -\dot{\pi}_S + \delta\pi_S = -\dot{\pi}_S + \delta A U_C \qquad \text{(A2.3)}$$

Using (A2.1), the time derivative of (A2.2) is

$$-\eta U_C\dot{C}/C - CU_{CS}/A - \dot{\pi}_S/A + \pi_S\dot{A}/A^2 = 0$$

which with (A2.2) and (A2.3) becomes

$$-\eta U_C\dot{C}/C - CU_{CS}/A + (U_S - \delta A U_C)/A + U_C\dot{A}/A = 0$$

$$\Rightarrow \dot{C} = [(CU_{CS} - U_S)/A - (\dot{A}/A - \delta)U_C]/(-\eta U_C/C), \quad \text{hence}$$

$$\dot{C} = [\dot{A}/A - \delta + (U_S - CU_{CS})/U_C A]C/\eta \qquad \text{(2.9)} \;\|$$

Appendix 2.2 *General cake-eating model: Derivation of the policy path (2.14)*

Taking growth rates of (2.12) and using (A2.1) gives

$$[-\eta U_C\dot{C}/C - CU_{CS}/A]/U_C = \dot{\pi}_S/\pi_S + \dot{\phi}_C/(1+\phi_C) - \dot{A}/A$$

which with (2.13) gives

$$[\eta U_C\dot{C}/C + CU_{CS}/A]/U_C = \dot{A}/A - \delta - \phi_S - \dot{\phi}_C/(1+\phi_C)$$

$$\dot{C} = [\dot{A}/A - \delta + \Phi - CU_{CS}/U_C A]C/\eta, \quad \text{where} \tag{2.14}$$

$$\Phi(t) := -\dot{\phi}_C/(1+\phi_C) - \phi_S. \tag{2.16}$$ ||

Appendix 2.3 *Special Case 1 of cake-eating model: Derivation of socially PV-optimal and free market paths (2.20)-(2.24)*

$$U = C^\nu S^\epsilon, \; A = e^{\tau t} \; \text{ in Special Case 1; hence} \tag{2.19}$$

$$\left.\begin{aligned} &U_C = \nu C^{\nu-1}S^\epsilon, \; U_S = \epsilon C^\nu S^{\epsilon-1}, \; U_{CS} = \nu\epsilon C^{\nu-1}S^{\epsilon-1} \\ &U_S/U_C = \epsilon C/\nu S, \; U_{CS}/U_C = \epsilon/S, \; U_{CC} = \nu(\nu-1)C^{\nu-2}S^\epsilon \\ &\eta = 1-\nu, \; \dot{A}/A = \tau \end{aligned}\right) \tag{A2.4}$$

From (2.2),

$$\left.\begin{aligned} &C = Re^{\tau t} \Rightarrow \dot{C}/C = \dot{R}/R + \tau = \ddot{S}/\dot{S} + \tau, \\ &C/S = -\dot{S}e^{\tau t}/S, \; \dot{U}/U = \nu(\ddot{S}/\dot{S}+\tau) + \epsilon\dot{S}/S \end{aligned}\right) \tag{A2.5}$$

Together (A2.4) and (2.9) give the *socially PV-optimal* path

$$(1-\nu)\dot{C} = [\tau - \delta + \epsilon(1-\nu)C/\nu Se^{\tau t}]C,$$

With (A2.5) this becomes

$$(1-\nu)(\ddot{S}/\dot{S}+\tau) = \tau - \delta - \epsilon(1-\nu)\dot{S}/\nu S.$$ ||

Try a solution $S = S_0 e^{-\theta t}$, which satisfies the initial condition. Inserting it into (2.21) it gives

$$(1-\nu)(\tau-\theta) = \tau - \delta - \epsilon(1-\nu)(-\theta)/\nu$$

$$\Rightarrow -[(1-\nu)+\epsilon(1-\nu)/\nu]\theta = \tau - \delta - \tau(1-\nu)$$

$$\Rightarrow [1+\epsilon/\nu](1-\nu)\theta = \delta - \tau\nu, \quad \text{hence}$$

$$\theta = \nu(\delta - \tau\nu)/[(1-\nu)(\nu+\epsilon)]. \tag{2.22}$$

Next, (2.22), (2.19) and (2.2) $\Rightarrow S = S_0 e^{-\theta t} \Rightarrow C = \theta S_0 e^{(\tau-\theta)t}$

$$\Rightarrow U = \theta^{\nu} S_0{}^{\nu+\epsilon} e^{[\tau\nu-\theta(\nu+\epsilon)]t} = \theta^{\nu} S_0{}^{(\nu+\epsilon)} e^{[\nu(\tau-\delta)/(1-\nu)]t} \tag{2.23}$$

since $\tau\nu-\theta(\nu+\epsilon) = [\tau\nu(1-\nu)(\nu+\epsilon)-\nu(\delta-\tau\nu)(\nu+\epsilon)] \,/\, [(1-\nu)(\nu+\epsilon)]$

$= [\tau\nu(1-\nu)-\nu(\delta-\tau\nu)] \,/\, (1-\nu) = \nu(\tau-\delta)/(1-\nu)$ ||

Now start from the general equation for the *free market* path:

$$\dot{C}^{\mu} = [\dot{A}/A-\delta-C^{\mu}U^{\mu}_{CS}/U^{\mu}_{C}A]C^{\mu}/\eta. \tag{2.17}$$

With (A2.4) this gives

$\dot{C} = (\tau-\delta-C\epsilon/SA)C/(1-\nu)$, which with (A2.5) gives

$\ddot{S}/\dot{S} + \tau = [\tau-\delta-\epsilon(-\dot{S}/S)]/(1-\nu)$

Inserting $S = S_0 e^{-\psi t}$, which satisfies the initial condition, then gives

$-\psi+\tau = (\tau-\delta-\epsilon\psi)/(1-\nu)$

$\Rightarrow \psi[\epsilon-(1-\nu)] = \tau[1-\nu-1]-\delta$

$\Rightarrow \psi = (\delta-\tau\nu)/(1-\nu-\epsilon)$,

which with (A2.5) gives

$\dot{U}/U = \nu[-(\delta-\tau\nu)/(1-\nu-\epsilon)+\tau] - \epsilon(\delta-\tau\nu)/(1-\nu-\epsilon)$

$= [-\nu(\delta-\tau\nu)+\tau\nu(1-\nu-\epsilon)-\epsilon(\delta-\tau\nu)]/(1-\nu-\epsilon)$

$= [-\nu\delta+\tau\nu-\epsilon\delta]/(1-\nu-\epsilon)$

$= \nu[\tau-\delta(\nu+\epsilon)/\nu]/(1-\nu-\epsilon)$ (2.24) ||

Appendix 2.4 Special Case 1 of cake-eating model: Proof that (2.27) is an opsustimal policy

The resource path corresponding to (2.27) is $S^{\dagger}=S_0 e^{-[\tau\nu/(\nu+\epsilon)]t}$, hence

$C^{\dagger}=[\tau\nu/(\nu+\epsilon)]S_0 e^{[\tau\epsilon/(\nu+\epsilon)]t}$ and

$\mathrm{PV}(U^{\dagger})=[\tau\nu/(\nu+\epsilon)]^{\nu} S_0{}^{\nu+\epsilon}/\delta$.

The only way that this path can be perturbed without causing a decline in utility (which would break the sustainability criterion) is for consumption to start out lower than $C^{\dagger}(0)$ and grow faster than $C^{\dagger}$ for some time. So look at a perturbed path (denoted ^{p}) defined by

$$S^{p} = S_0 e^{-(\kappa-a)t},\ 0 \leq t \leq b,\ \text{where } \kappa = \tau\nu/(\nu+\epsilon) \text{ and } 0 < a,\ b \ll 1;$$
$$= S_0 e^{ab} e^{-\kappa t},\quad b \leq t \leq \infty.$$

The PV of this path is

$$\mathrm{PV}(U^{p}) = (\kappa-a)^{\nu} S_0^{\nu+\epsilon}[1-e^{-[\delta-a(\nu+\epsilon)]b}]/[\delta-a(\nu+\epsilon)] +$$
$$\kappa^{\nu} S_0^{\nu+\epsilon} e^{-[\delta-a(\nu+\epsilon)]b}/\delta$$

$$\Rightarrow \qquad \partial\mathrm{PV}(U^{p})/\partial b \propto + \ (\kappa-a)^{\nu} e^{-[\delta-a(\nu+\epsilon)]b} - [\delta-a(\nu+\epsilon)]\kappa^{\nu} e^{-[\delta-a(\nu+\epsilon)]b}/\delta$$
$$\propto + \ (1-a/\kappa)^{\nu} - [1-a(\nu+\epsilon)/\delta]$$
$$\approx 1 - \nu a/\kappa - 1 + a(\nu+\epsilon)/\delta \quad \text{for } a \text{ small}$$
$$\propto + \ -(\nu+\epsilon)/\tau + (\nu+\epsilon)/\delta < 0$$

(since we assume $\delta > \tau$ for there to be a need for sustainability policy in the first place). Since the PV of the perturbed path thus decreases as the length of its perturbed path increases, the PV of the original unperturbed path is the maximum that can be attained. ||

Appendix 2.5 Special Case 1 of cake-eating model: Proof that social sustainability costs less than individual sustainability

Here we distinguish between the individual's resource stock s (which determines her consumption c) and the social resource stock S (which directly enters her utility function $u = c^{\nu}S^{\epsilon}$). On a socially just sustained path, the individual resource stock is $s^{\dagger} = s_0 e^{-\kappa t}$ where $\kappa = \psi(\Phi^{\dagger}) = \tau\nu/(\nu+\epsilon)$ from (2.27) and the social resource stock is $S^{\dagger} = S_0 e^{-\kappa t}$, so from (2.1) and (2.25),

$$u^{\dagger} = (\kappa s_0 e^{(\tau-\kappa)t})^{\nu}(S_0 e^{-\kappa t})^{\epsilon} = \kappa^{\nu} s_0^{\ \nu} S_0^{\ \epsilon} \quad \text{and} \quad \mathrm{PV}(u^{\dagger}) = \kappa^{\nu} s_0^{\ \nu} S_0^{\ \epsilon}/\delta.$$

On the individually sustainable path (denoted ^{i}), individual resource stock and consumption are (with $\zeta = \tau - \epsilon\theta/\nu$):

$$s^i = s_0 e^{-\zeta t} \Rightarrow c^i = \zeta s_0 e^{(\tau-\zeta)t}$$

but the social resource stock is the socially PV-optimal $\widetilde{S}$, so utility is

$$u^i = (c^i)^\nu(\widetilde{S})^\epsilon = (\zeta s_0 e^{(\tau-\zeta)t})^\nu(S_0 e^{-\theta t})^\epsilon = \zeta^\nu s_0{}^\nu S_0{}^\epsilon,$$

$$\Rightarrow \quad \mathrm{PV}(u^i) = \zeta^\nu s_0{}^\nu S_0{}^\epsilon/\delta$$

$$\Rightarrow \quad \mathrm{PV}(u^i)-\mathrm{PV}(u^\dagger) \propto+ \zeta^\nu-\kappa^\nu.$$

Now $\quad \zeta = \tau-\epsilon\theta/\nu = [\tau\nu(1-\nu)-(\delta-\tau)\epsilon]/(1-\nu)(\nu+\epsilon)$

so $\quad \zeta-\kappa = [\tau\nu(1-\nu)+(\delta-\tau)\epsilon-\tau\nu(1-\nu)]/(1-\nu)(\nu+\epsilon)$

$\quad = (\tau-\delta)\epsilon/(1-\nu)(\nu+\epsilon)<0$ since $\delta>\tau$ by assumption and $0<\nu<1$.

Hence $\mathrm{PV}(u^i) < \mathrm{PV}(u^\dagger)$. ||

Appendix 2.6 Special Case 2 of cake-eating model: Derivation of socially PV-optimal and policy paths (2.29)-(2.30)

$$U = \nu\log C + \epsilon\log S \quad \text{and} \quad A = (1+\tau t)^2 \tag{2.28}$$

gives $\quad U_C=\nu/C,\ U_S=\epsilon/S,\ U_{CS}=0,\ \dot{U}=\nu\dot{C}/C+\epsilon\dot{S}/S,\ \eta=1,$) (A2.6)

$\quad \dot{U}_C/U_C=-\dot{C}/C,\ \dot{C}/C=2\tau/(1+\tau t)+\ddot{S}/\dot{S},\ \dot{A}/A=2\tau/(1+\tau t)$)

From (2.28) and (2.2), the undiscounted Hamiltonian is

$$H = U(C,S) + \pi_S\dot{S} = U(C,S) - \pi_S C/(1+\tau t)^2$$

with a first order condition $\partial H/\partial C = U_C-\pi_S/(1+\tau t)^2 = 0$, which gives

$$\pi_S = (1+\tau t)^2 U_C \tag{A2.7}$$

Taking growth rates of (A2.7) and using (A2.6) gives

$$\dot{\pi}_S/\pi_S = 2\tau/(1+\tau t) + \dot{U}_C/U_C = 2\tau/(1+\tau t) - \dot{C}/C = -\ddot{S}/\dot{S} \tag{A2.8}$$

On the socially PV-optimal path,

$$\partial H/\partial S = -\dot{\pi}_S+\delta\pi_S = U_S \Rightarrow \dot{\pi}_S/\pi_S = \delta-U_S/\pi_S = \delta-U_S/(1+\tau t)^2 U_C$$

which using (A2.8) and (A2.6) gives

$$-\ddot{S}/\dot{S} = \delta - \epsilon C/\nu S(1+\tau t)^2 = \delta + (\epsilon/\nu)\dot{S}/S$$

Try $S = S_0 e^{-\psi t}$

$\Rightarrow \psi = \delta - \epsilon\psi/\nu, \quad \psi = \delta\nu/(\nu+\epsilon)$, and

$$S = S_0 e^{-\delta\nu t/(\nu+\epsilon)} \tag{2.29}$$

$$C = (1+\tau t)^2[\delta\nu/(\nu+\epsilon)]S_0 e^{-\delta\nu t/(\nu+\epsilon)}$$

$$U = \nu\log[(1+\tau t)^2\delta\nu/(\nu+\epsilon)] + (\nu+\epsilon)\log[S_0 e^{-\delta\nu t/(\nu+\epsilon)}]$$

$$= 2\nu\log(1+\tau t) - \delta\nu t + \nu\log[\delta\nu/(\nu+\epsilon)] + (\nu+\epsilon)\log S_0$$

$$\dot{U} = 2\tau\nu/(1+\tau t) - \delta\nu$$

Inserting (A2.6) into the general policy path gives

$$\dot{C} = [\dot{A}/A - \delta + \Phi - CU_{CS}/U_C A]C/\eta \tag{2.13}$$

$$\Rightarrow \dot{C}/C = 2\tau/(1+\tau t) - \delta + \Phi = 2\tau/(1+\tau t) + \ddot{S}/\dot{S}$$

$\Rightarrow S = S_0 e^{-(\delta-\Phi)t}$ is the solution, by inspection (2.30) ||

$$\Rightarrow C = (1+\tau t)^2(\delta-\Phi)S_0 e^{-(\delta-\Phi)t};$$

$$\Rightarrow U = 2\nu\log(1+\tau t) - (\nu+\epsilon)(\delta-\Phi)t + \nu\log(\delta-\Phi) + (\nu+\epsilon)\log S_0.$$

Appendix 2.7 Special Case 2 of cake-eating model: Derivation of constant utility path (2.31)

Set $\dot{U}$ from (A2.6), to zero:

$$\dot{U} = \nu[2\tau/(1+\tau t) + \ddot{S}/\dot{S}] + \epsilon\dot{S}/S = 0$$

Try $S = S_0(1+\tau t)^{-\psi}$

$$\Rightarrow \dot{S} = -\psi\tau S_0(1+\tau t)^{-\psi-1}, \quad \ddot{S} = (\psi+1)\psi\tau^2 S_0(1+\tau t)^{-\psi-2},$$

$$\dot{S}/S = -\psi\tau/(1+\tau t), \quad \ddot{S}/\dot{S} = -(\psi+1)\tau/(1+\tau t)$$

$$\Rightarrow \nu[2\tau - (\psi+1)\tau] - \epsilon\psi\tau = 0$$

$$\Rightarrow (\epsilon+\nu)\psi\tau = \nu(2\tau-\tau)$$

$\Rightarrow \psi = \nu/(\nu+\epsilon), \quad S = S_0(1+\tau t)^{-\nu/(\nu+\epsilon)}$ (2.31) ||

Appendix 2.8 General capital-resource model: Derivation of socially PV-optimal path (2.34)–(2.36)

The derivation closely follows Krautkraemer (p159). The undiscounted Hamiltonian for an interior solution of the maximisation problem (2.1) subject to conditions (2.2)-(2.5) and (2.32) is

$$H = U(C,S) + \pi_K[AF(K,R)-C] - \pi_S R$$

where $\pi_K(t)$ and $\pi_S(t)$ are the undiscounted shadow prices of the capital and resource stocks respectively. We will use the linear homogeneity of the production function to write it in the intensive form, $F(K/R,1) = f(x)$, whence (see Dasgupta and Heal 1974, p11)

$$F_K=f', \; F_R=f-xf', \; \dot{F}_R=-x\dot{x}f'', \; F_R=f-xf'=-\sigma xff''/f' \qquad \text{(A2.9)}$$

where σ is the elasticity of substitution. The first order conditions are

$$\partial H/\partial C = 0 \quad \Rightarrow \quad \pi_K = U_C \qquad \text{(A2.10)}$$

$$\partial H/\partial R = 0 \quad \Rightarrow \quad \pi_S = \pi_K AF_R = U_C AF_R \qquad \text{(A2.11)}$$

$$\partial H/\partial K = -\dot{\pi}_K + \delta\pi_K = \pi_K AF_K \quad \Rightarrow \quad \dot{\pi}_K = (\delta - AF_K)U_C \qquad \text{(A2.12)}$$

$$\partial H/\partial S = -\dot{\pi}_S + \delta\pi_S = U_S \quad \Rightarrow \quad \dot{\pi}_S = \delta U_C AF_R - U_S \qquad \text{(A2.13)}$$

Differentiating (A2.10) with respect to time and equating with (A2.12) gives

$$(\delta - AF_K)U_C = U_{CC}\dot{C} - U_{CS}R$$

$$\Rightarrow -(U_{CC}/U_C)\dot{C} = AF_K - \delta - R(U_{CS}/U_C),$$

$$\Rightarrow \dot{C}/C = [Af - \delta - R(U_{CS}/U_C)]/\eta \qquad \text{(2.34)} \;\|$$

There is no U_S term in (2.34), so it holds for both social-PV and private-PV cases, and the expression for the rate of change of utility over time is

$$\dot{U} = U_C\dot{C} - U_S R = U_C C(Af' - \delta)/\eta - R(CU_{CS} + \eta U_S)/\eta \qquad \text{(2.35)} \;\|$$

Taking growth rates over time from (A2.11) gives

$$\dot{\pi}_S/\pi_S = \dot{\pi}_K/\pi_K + \dot{A}/A + \dot{F}_R/F_R$$

Substituting from (A2.10)-(A2.13) in the socially PV-optimal case gives

$$(\delta U_C AF_R - U_S)/U_C AF_R = \delta - AF_K + \dot{A}/A + \dot{F}_R/F_R$$

$$\Rightarrow\ -U_S/U_C A F_R \;=\; -AF_K + \dot{A}/A + \dot{F}_R/F_R \qquad \text{(A2.14)} \;\|$$

which with (A2.9) gives

$$U_S/U_C A(\sigma x f f''/f') \;=\; -Af' + \dot{A}/A + x\dot{x}f''/(\sigma x f f''/f')$$

$$\Rightarrow\ U_S/U_C A\sigma x f f'' \;=\; -A + \dot{A}/Af' + \dot{x}/\sigma f, \text{ hence}$$

$$\dot{x}/x = (A - \dot{A}/Af')\sigma f/x \;-\; U_S/U_C A x^2(-f'') \qquad (2.36)$$

which reduces to (2.21) in Krautkraemer (1985) when $A = 1$. ||

Appendix 2.9 General capital-resource model: Derivation of footnote 20 (Hartwick's Rule)

If there is no technical progress ($A=1$), the modified Hotelling Rule for Pareto-efficient resource depletion (A2.14) reads as

$$-U_S/U_C F_R \;=\; -F_K + \dot{F}_R/F_R$$

$$\Rightarrow\ U_C\dot{F}_R = U_C F_K F_R \;-\; U_S \qquad \text{(A2.15)}$$

Combined with the Hartwick Rule, $\dot{K} = RF_R$, this gives constant utility:

$$\begin{aligned}
\dot{U} &= U_C(d/dt)(F-\dot{K}) \;-\; RU_S && \text{using (2.32)}\\
&= U_C(d/dt)(F-RF_R) \;-\; RU_S && \text{using } \dot{K}=RF_R\\
&= U_C(\dot{F}-\dot{R}F_R) - R(U_C F_K F_R - U_S) \;-\; RU_S && \text{using (A2.15)}\\
&= U_C(\dot{F}-\dot{R}F_R-\dot{K}F_K) && \text{using (A2.15) and } RF_R=\dot{K}\\
&= 0
\end{aligned}$$

||

Appendix 2.10 General capital-resource model: Derivation of the policy path (2.37)-(2.39)

To save tedious repetition, we omit the hat ($\hat{}$) overscripts here. Specific taxes are paid (in units of the consumption good) at a rate ϕ_C on consumption, ϕ_K on the capital stock, ϕ_C on resource depletion and ϕ_S on the resource stock, and revenue-neutrality is achieved by lump sum refunds Ω. From the point of view of the individual, the output budget constraint now includes tax payments as

$$Q = C + \dot{K} + \phi_C C + \phi_K K + \phi_R R + \phi_S S - \Omega.$$

The undiscounted Hamiltonian for the policy problem is then

$$H = U(C,S) + \pi_K[AF(K,R) - (1+\phi_C)C - \phi_K K - \phi_R R - \phi_S S + \Omega] - \pi_S R$$

with first order conditions

$$\partial H/\partial C = 0 \quad \Rightarrow \quad \pi_K(1+\phi_C) = U_C \qquad \text{(A2.16)}$$

$$\partial H/\partial R = 0 \quad \Rightarrow \quad \pi_S = \pi_K(AF_R - \phi_R) = U_C(AF_R - \phi_R)/(1+\phi_C) \qquad \text{(A2.17)}$$

$$\partial H/\partial K = -\dot{\pi}_K + \delta\pi_K = \pi_K(AF_K - \phi_K) \qquad \text{(A2.18)}$$

$$\Rightarrow \quad \dot{\pi}_K = (\delta + \phi_K - AF_K)U_C/(1+\phi_C)$$

$$\partial H/\partial S = -\dot{\pi}_S + \delta\pi_S = -\pi_K\phi_S \qquad \text{(A2.19)}$$

(U_S is the external amenity cost of resource depletion, and therefore ignored by a private agent in arriving at (A2.19)). Taking the time derivative of (A2.16) and using (A2.18) and $\dot{U}_C = -\eta U_C \dot{C}/C - CU_{CS}/A$ gives

$$-\eta U_C \dot{C}/C - RU_{CS} = \dot{\pi}_K(1+\phi_C) + \pi_K\dot{\phi}_C$$
$$= (\delta + \phi_K - AF_K)U_C + U_C\dot{\phi}_C/(1+\phi_C)$$

$$\Rightarrow \qquad \eta\dot{C}/C + RU_{CS}/U_C = AF_K - \delta - \dot{\phi}_C/(1+\phi_C) - \phi_K$$

$$\Rightarrow \qquad \dot{C}/C = [AF_K - \delta - \dot{\phi}_C/(1+\phi_C) - \phi_K - RU_{CS}/U_C]/\eta \qquad \text{(2.37)}$$

and (2.38) for $\dot{U}$ follows from (2.37) and (2.6).

The growth rate of (A2.17) gives

$$\dot{\pi}_S/\pi_S = \dot{\pi}_K/\pi_K + (\dot{A}F_R + A\dot{F}_R - \dot{\phi}_R)/(AF_R - \phi_R)$$

which with (A2.17), (A2.18) and (A2.19) gives

$$\delta + \pi_K\phi_S/\pi_S = \delta + \phi_K - AF_K + (\dot{A}F_R + A\dot{F}_R - \dot{\phi}_R)/(AF_R - \phi_R)$$
$$= \delta + \phi_S/(AF_R - \phi_R)$$

$$\Rightarrow \qquad \phi_K - AF_K + (\dot{A}F_R + A\dot{F}_R - \dot{\phi}_R)/(AF_R - \phi_R) = \phi_S/(AF_R - \phi_R).$$

Substituting $F_K = f'$, $F_R = f - xf'$, $\dot{F}_R = -x\dot{x}f''$ and $F_R = f - xf' = -\sigma x f f''/f'$ where σ is the elasticity of substitution (see Dasgupta and Heal 1974, p11) gives

$$\phi_K - Af' + [\dot{A}(f - xf') + A(-x\dot{x}f'') - \dot{\phi}_R - \phi_S]/[A(f - xf') - \phi_R] = 0$$

$$\Rightarrow A\dot{x}/x = (\phi_K - Af')[A(f - xf') - \phi_R]/x^2 f'' + [-\dot{A}\sigma f/xf' - \dot{\phi}_R/x^2 f'' - \phi_S/x^2 f'']$$

$$= -(\phi_K - Af')A\sigma f/xf' - [(\phi_K - Af')\phi_R + \dot{\phi}_R + \phi_S]/x^2 f'' - \dot{A}\sigma f/xf'$$

$$\Rightarrow \dot{x}/x = [A - (\dot{A}/A + \phi_K)/f']\sigma f/x + \Phi_x/x^2 f'', \tag{2.39}$$

where $\Phi_x := f'\phi_R - (\dot{\phi}_R + \phi_S + \phi_K\phi_R)/A.$ ||

Appendix 2.11 Special case of capital-resource model: Derivation of socially PV-optimal growth rates (2.41)

From (2.40),

$$\left.\begin{array}{l} A = e^{\tau t}, \quad \dot{A}/A = \tau, \quad f(x) = x^\alpha, \quad \sigma = 1, \quad f' = \alpha x^{\alpha-1}, \\ -x^2 f'' = \alpha(1-\alpha)x^\alpha, \quad \alpha r = \alpha A x^{\alpha-1} = Af' \end{array}\right) \tag{A2.20}$$

Inserting this and (A2.4) into (2.34) and (2.36) respectively gives the first two equations of set (2.41). Taking growth rates of (2.40) gives the third. ||

Appendix 2.12 *Special case of capital-resource model: Derivation of socially PV-optimal growth rates in steady state (2.42)-(2.45)*

Inserting

$$\left.\begin{aligned} &\dot{C}/C=\dot{K}/K=\dot{Q}/Q=\xi,\ Q/K=r/\alpha,\ R/S=-\dot{R}/R=\gamma \\ &\dot{x}/x = \dot{K}/K-\dot{R}/R = \xi+\gamma \\ &e^{\tau t}/x^{1-\alpha} = K^{\alpha}R^{1-\alpha}e^{\tau t}/K = Q/K = r/\alpha \\ &C/Se^{\tau t}x^{\alpha} = (Q-\dot{K})R/Se^{\tau t}x^{\alpha-1}K = (r\alpha-\mu)\gamma/r\alpha \end{aligned}\right) \quad \text{(A2.21)}$$

into (2.41) gives

$$(1-\nu)\xi = r - \delta - \epsilon\gamma \tag{A2.22}$$

$$\xi+\gamma = r - \tau/\alpha - [\epsilon/\nu\alpha(1-\alpha)](r\alpha-\mu)\gamma\alpha/r \tag{A2.23}$$

$$\xi = \alpha\xi - (1-\alpha)\gamma + \tau, \text{ giving} \tag{A2.24}$$

$$\xi = \tau/(1-\alpha) - \gamma \tag{2.44}$$

which with $r = (1-\nu)\xi + \delta + \epsilon\gamma$ from (A2.22) gives

$$r = (1-\nu)\tau/(1-\alpha) + \delta - (1-\nu-\epsilon)\gamma \tag{2.43}$$

(A2.23), (2.43) and (2.44) together give

$$\gamma = [\delta-\tau\nu/(1-\alpha)]/(1-\nu-\epsilon) - \epsilon(r/\alpha-\xi)\gamma\alpha/\nu(1-\alpha)(1-\nu-\epsilon)r \tag{2.42}$$

and (2.40) and (2.44) give

$$\dot{U}/U = \nu\xi - \epsilon\gamma = \tau\nu/(1-\alpha) - (\nu+\epsilon)\gamma. \tag{2.45}$$ ||

Appendix 2.13 *Special case of capital-resource model: Derivation of policy path in steady state (2.47)*

Inserting (A2.20) and (A2.4) into (2.37) and (2.39), and merely copying the last equation of set (2.41) respectively gives

$$(1-\nu)\dot{C}/C = e^{\tau t}\alpha x^{\alpha-1} - \delta + \Phi_C - \phi_K - \epsilon R/S$$

$$\dot{x}/x = [e^{\tau t}-(\tau+\phi_K)/\alpha x^{\alpha-1}]x^{\alpha-1} - \Phi_x/\alpha(1-\alpha)x^{\alpha}$$

$$\dot{Q}/Q = \alpha\dot{K}/K + (1-\alpha)\dot{R}/R + \tau.$$

Inserting (A2.21) and (A2.4) into these then gives

$$(1-\nu)\xi = r - \delta + \Phi_C - \phi_K - \epsilon\gamma \quad \text{and}$$
$$\xi+\gamma = (r-\tau-\phi_K)/\alpha - \Phi_x/\alpha(1-\alpha)x^\alpha; \quad \text{and}$$
$$\xi = \tau/(1-\alpha) - \gamma \ \text{ is unchanged, from (2.44).}$$

These are three simultaneous linear equations in γ, r and ξ, with solution

$$\gamma = [\delta-\Phi_C-\Phi_x/(1-\alpha)x^\alpha-\tau\nu/(1-\alpha)]/(1-\nu-\epsilon) \tag{2.47}$$
$$r = \tau/(1-\alpha) + \phi_K + \Phi_x/(1-\alpha)x^\alpha$$
$$\xi = \{\tau(1-\epsilon)/(1-\alpha)-[\delta-\Phi_C-\Phi_x/(1-\alpha)x^\alpha]\}/(1-\nu-\epsilon)$$

and from the first equality in (2.45),

$$\dot{U}/U = \{\tau\nu/(1-\alpha)-(\nu+\epsilon)[\delta-\Phi_C-\Phi_x/(1-\alpha)x^\alpha]\}/(1-\nu-\epsilon) \qquad \|$$

Appendix 2.14 Special case of capital-resource model, with diminishing returns: Derivation of interest rate in steady state (2.49)

The Ramsey rule for the steady state policy path of the diminishing returns economy can be obtained from (2.37) to give the same equation as in the constant returns to scale case:

$$(1-\nu)\xi = r - \delta + \Phi_C - \phi_K - \epsilon\gamma$$

From (2.48), the steady state growth accounting relationship is different from the constant returns case, being

$$\xi = \tau + \alpha\xi - \pi\gamma \Rightarrow (1-\alpha)\xi + \pi\gamma = \tau$$

The Hotelling Rule in its basic form (the rate of interest equals the return to holding the resource as its competitive price rises) applies to the free market path where the resource's amenity value is not reflected in its price. A conservation policy such as a declining resource tax or resource stock subsidy, say of overall strength Φ_Σ (not the same as Φ_x, since (2.39) does not hold in this case), will add to the return of holding the resource. The modified Hotelling Rule for the policy path is then, using (2.48):

$$r = \dot{Q}_R/Q_R + \Phi_\Sigma = \tau + \alpha\xi + (\pi-1)(-\gamma) + \Phi_\Sigma$$

As in Appendix 2.14, we have now three simultaneous linear equations in

γ, r and ξ, whose solution can be computed as

$$\gamma = [(\delta-\Phi_C+\phi_K-\Phi_\Sigma)(1-\alpha)-\tau\nu] \;/\; [(1-\epsilon)(1-\alpha)-\pi\nu]$$

$$r = \{(\delta-\Phi_C+\phi_K)(1-\alpha-\pi)+\tau(1-\epsilon-\nu)+\Phi_\Sigma[\pi(1-\nu)-\epsilon(1-\alpha)]\} \;/\; [(1-\epsilon)(1-\alpha)-\pi\nu] \tag{2.49}$$

$$\xi = \tau(1-\epsilon)-\pi(\delta-\Phi_C+\phi_K-\Phi_\Sigma) \;/\; [(1-\epsilon)(1-\alpha)-\pi\nu], \quad \text{and hence}$$

$$\dot{U}/U = \{\tau\nu-(\delta-\Phi_C+\phi_K-\Phi_\Sigma)[\pi\nu+\epsilon(1-\alpha)]\} \;/\; [(1-\epsilon)(1-\alpha)-\pi\nu]$$

so $\quad \delta-\Phi_C+\phi_K-\Phi_\Sigma<0$ improves sustainedness. ||

Appendix 2.15 Special case of capital-resource model: Approach to steady state

In the non-environmental model in Stiglitz (1974), one could conclude that the combination of a sustainable steady state ($\dot{\widetilde{C}}^*/\widetilde{C}^*$, and hence $\dot{\widetilde{U}}^*/\widetilde{U}^*$ > 0), and an initial endowment of capital and resource which is "capital-poor" (or "resource-rich", which is the same thing) relative to the steady state, would guarantee that the PV-optimal approach to the steady state would be sustained. This is because from (2.41), PV-optimal utility growth in Stiglitz' model is

$$\dot{\widetilde{U}}/\widetilde{U} = \nu\dot{\widetilde{C}}/\widetilde{C} = \nu[e^{\tau t}\alpha(R/K)^{1-\alpha}-\delta]/(1-\nu)$$

and resource-richness means a higher $\widetilde{R}/\widetilde{K}$ during the approach than at the steady state, and hence a higher rate of interest and utility growth. But equations (2.6) and (2.41) for socially PV-optimal growth in our double Cobb-Douglas case with a resource amenity give

$$\dot{\widetilde{U}}/\widetilde{U} = \nu[e^{\tau t}\alpha(\widetilde{R}/\widetilde{K})^{1-\alpha}-\delta-\epsilon\widetilde{R}/\widetilde{S}]/(1-\nu) \;-\; \epsilon\widetilde{R}/\widetilde{S}$$

and resource-richness means lower $\widetilde{K}/\widetilde{R}$ (and hence higher $\dot{\widetilde{U}}/\widetilde{U}$) again, but also higher $\widetilde{R}/\widetilde{S}$ (and hence lower $\dot{\widetilde{U}}/\widetilde{U}$). The net effect on $\dot{\widetilde{U}}/\widetilde{U}$ is ambiguous, so unsustainability cannot be ruled out on any PV-optimal approach to a steady-state equilibrium, and a complex (non-asymptotic) policy intervention might be required to achieve a sustained path. ||

Figure 2.1 Description and notation of development paths

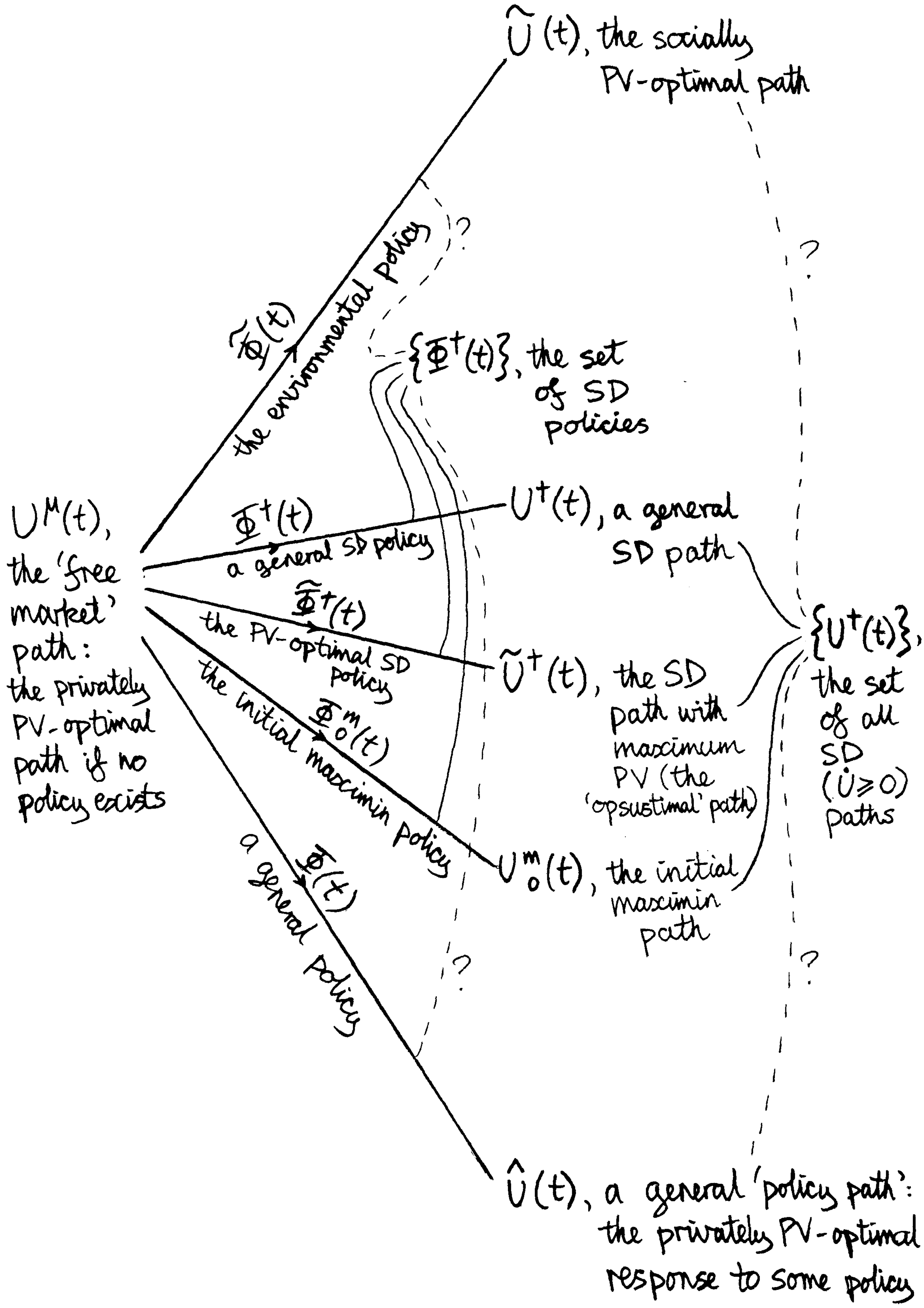


Figure 2.2 Resolution of two apparent problems with the non-declining utility criterion

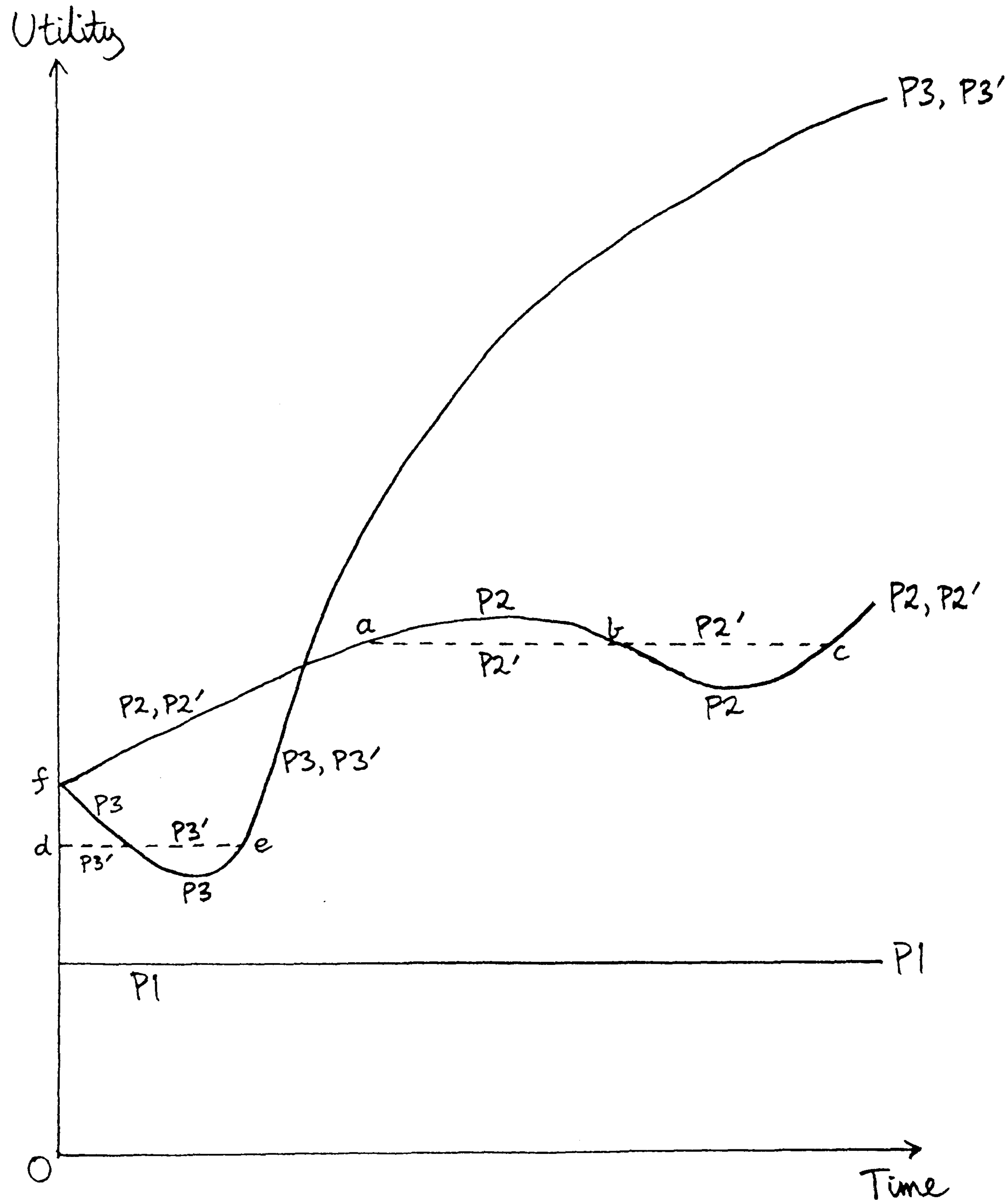
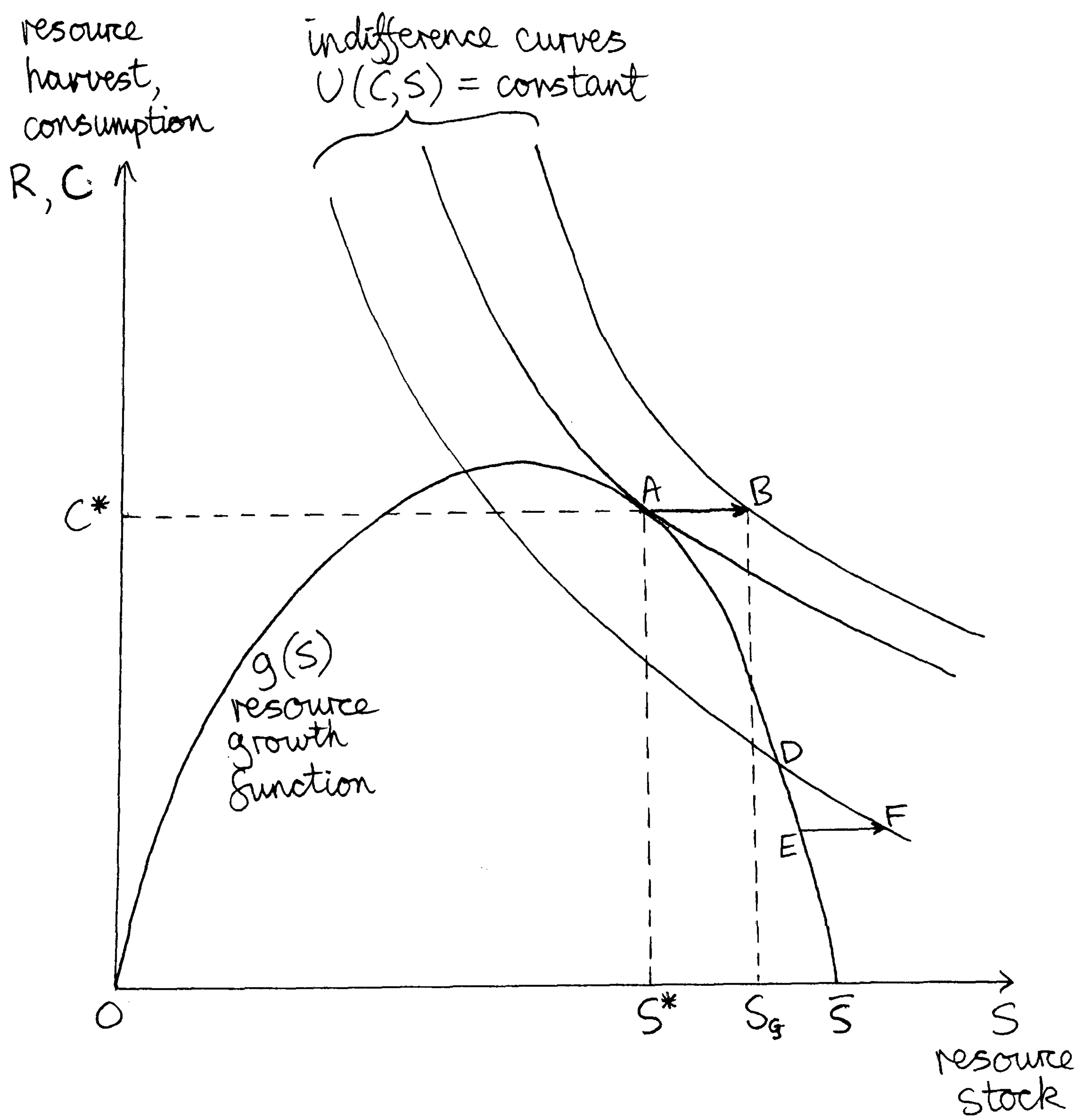


Figure 2.3 How a higher initial stock of renewable resources can harm sustainability



CHAPTER 3

THE OPTIMAL SUSTAINABLE DEPLETION OF NON-RENEWABLE RESOURCES*

3.1 INTRODUCTION

> "Of course, later generations (should they exist) suffer incredibly as a result of the initial profligacy under the Utilitarian programme. They are far worse off than they would be under the maxi-min policy." (Dasgupta and Heal 1979, p299)

> "...the maxi-min criterion is at the mercy of initial conditions. An economy wedded to it is imprisoned in poverty should it have been unfortunate enough to have inherited low stocks of capital and resources from the past." (Dasgupta and Heal 1979, p310)

What is a fair and sensible way to use up the finite, non-renewable resources of the natural world over time? Should we maximise the Utilitarian objective of the present discounted value (PV) of utility? Or should we maximise the utility of the worst-off generation? The above quotations highlight the dilemma that society faces if it has to choose solely between the Utilitarian criterion (hereafter called *PV-optimality*) and the maximin criterion, in an economy where output can be maintained only by substituting an ever-increasing stock of human-made capital for ever-

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declining inputs of non-renewable resources.[1,2] By many people's standards, the 'incredible suffering' that PV-optimality can visit on later generations is not fair. But neither is the maximin criterion very sensible, because it bans any development which can raise welfare over time, even when initial generations would gladly consume less in order to make the necessary investments (Solow 1974). So would an alternative criterion for intertemporal equity, namely to maximise PV subject to a *sustainedness* constraint,[3] resolve this somewhat? One expects that this would allow early development, while protecting distant generations against suffering, and so may offer a useful compromise between the PV-optimal and maximin criteria.

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1. Despite considerable doubts about whether the world economy can be run on a litre of oil, I assume throughout this chapter that capital is endlessly substitutable for any natural resource in the production process.

2. In recent years the rate of speciation of terms in resource economics has greatly exceeded the rate of extinction, and the resulting terminological diversity requires me to make my meanings clear. Where possible I use *capital* only for durable, productive assets produced by people, *resources* for durable, productive assets found in nature, and *wealth* for the combined power of the two to give utility. This follows the dominant definition of capital as "a factor of production produced by the economic system" (Pearce 1992), and suits my purposes better than the corresponding terms 'human-made capital', 'natural capital' and 'aggregate capital'. But the latter do make sense if one takes the broader view that "The essence of capital, therefore, is that it represents deferred consumption" (Bannock, Baxter and Davis 1992). *Investment* here refers solely to changes in capital net of depreciation, while *aggregate investment* means the inner product of the rates of change of all capital and resource stocks with their respective prices. *Aggregate wealth* is the time integral of aggregate investment.

3. Recall that sustainedness is a synonym for sustainable development (SD) in this thesis. For no particular reason, the former phrase is mainly used in this Chapter, whereas the latter was mainly used in Chapter 2.

A major purpose of this chapter is to see if these expectations are correct, using a formal definition of sustainedness as a future in which a representative member of society has permanently *non-declining utility* (NDU). I have discussed the political and philosophical basis of this definition at length in Pezzey (1989, 1992) and in Chapters 1 and 2. My conclusion from those discussions is that while by no means ideal, NDU is a criterion of intergenerational equity with fewer drawbacks and more advantages than commonly supposed, and is therefore worth exploring. Chapter 2 also explored some of its implications in models quite similar to the one used here; but the main emphasis there was on the distinction between sustainedness and environmental policies in the presence of cumulative externalities. Here we abstract from any externalities, and focus on the properties of the PV-optimal sustained path, which I call the *opsustimal* path. Utility is an increasing function of consumption alone, so sustainedness can be equated with either non-declining utility forever or non-declining consumption forever, whichever is more convenient at the time.

Opsustimality is, however, not the only issue considered in the chapter. An informal introduction to and motivation for what follows is given by **Figure 3.1**.[4] We will assume a competitive 'capital-resource' economy, defined as one with reproducible capitals and finite, non-renewable resources where agents take prices as given; and three further assumptions about this economy are built into the Figure. (1) The PV-optimal path of utility in the economy will follow a *single-peaked* time path like $\widetilde{U}(t)$, which

4. Some of the notation in the Figure may seem a trifle excessive at this stage, but it is used for consistency with what appears later. Also, some of its defining features will not be needed for the first four propositions derived below, which are true more generally.

rises to a peak at time T_P and declines thereafter.[5] (2) The structure of production and utility in the economy allows a level of constant utility above some minimum tolerable level to be sustained forever. (3) The initial PV-optimal utility level $\widetilde{U}(0)$ is sustainable because it is less than U_0^m, the maximin (i.e. maximum sustainable) utility level starting from time zero.

To achieve a sustained path in such an economy, it will clearly be no use waiting until the peak time T_P when utility actually starts declining before intervening with policy action, as Pearce (1993, p48) effectively pointed out, since the peak utility level $\widetilde{U}(T_P)$ is obviously unsustainable. To see this, define $\widetilde{U}_P^m$, the *current maximin utility* at T_P, as the maximin utility level starting from the capital and resource stocks existing on the PV-optimal path at T_P. Then $\widetilde{U}_P^m$ must be below $\widetilde{U}(T_P)$, or the falling path of $\widetilde{U}$ after T_P could not be PV-optimal. Section 3.2 therefore addresses the question: How can we tell if the current PV-optimal utility level is still sustainable? In particular, assuming by continuity that the latest time when it is sustainable is T_L, the time when the current maximin utility level $\widetilde{U}_L^m$ equals PV-optimal utility $\widetilde{U}(T_L)$, is there some measurable condition or *predictor* of sustainability, such as non-declining aggregate wealth in the economy, which will be positive before T_L and negative after T_L?

These questions are interesting in their own right. Answering them obviously provides examples of sustained paths, such as staying at U_0^m forever, or following $\widetilde{U}(t)$ to T_L and then staying at $\widetilde{U}_L^m$ forever. But this tells us little about the opsustimal path. Neither, at first blush, will Section 3.3, which backtracks to find the conditions for the single-peakedness and initial sustainability properties of Figure 3.1 to hold true. But it turns out

5. A single-peaked path is rising for a finite time and then always falling (declining).

that these conditions greatly help the analysis of opsustimality in a *Dasgupta-Heal economy*, which I define as an economy with one capital stock, one non-renewable resource stock, and no technical change (as in the first model in Dasgupta and Heal 1974, hereafter **DH74**). Section 3.4 takes the analysis of opsustimality in a fairly general Dasgupta-Heal economy as far as possible, and Section 3.5 finds further results (both analytic and numerical) for a special case where the PV-optimal path can be calculated analytically. Section 3.6 briefly reviews what policies might achieve a sustained path in the general Dasgupta-Heal economy, following the methods established in Chapter 2. Section 3.7 concludes.

This analysis draws heavily on the classics of the resource economics and national accounting literatures: on DH74, who showed how the PV-optimal depletion of a non-renewable resource can lead to the asymptotic misery shown on Figure 3.1; on Solow (1974), who showed the possible existence of a constant, positive consumption path which would be selected by the maximin criterion; on Weitzman (1976), who showed that net national product (consumption plus aggregate investment) equals the consumption level which, if held constant, has the same PV as the PV-optimal path; on Hartwick (1977), who showed that zero aggregate investment (that is, diverting all the rents from resource depletion into capital investment) achieves constant consumption; and on Solow (1986), who effectively claimed that if an appropriate measure of aggregate wealth[6] is non-declining now, then declining utility can be avoided forever. I have also recycled,

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6. There is no entirely suitable term for this concept, which is formally defined below as an integral of aggregate investment. Solow (1986) used 'appropriate stock of capital', Solow (1993) used 'total stock of capital', 'broad stock of capital' or 'aggregate capital'; Pearce, Markandya and Barbier (1989) simply used 'wealth'; and Pearce et al (1993) used 'aggregate capital stock'. I prefer 'wealth' to 'capital' for reasons explained in footnote 2.

independently discovered, and/or extended several results from Asheim's (1988, 1994) work on intertemporal resource allocation, and on net national product as an indicator of sustainability.

3.2 AGGREGATE WEALTH AND SUSTAINABILITY

This section takes up the question: How can we tell if the current level of utility in the economy is sustainable? In particular, does non-declining aggregate wealth in the economy guarantee sustainability? Owing to a confusion that has arisen in the literature on national income accounting, particularly from the seminal contributions by Weitzman (1976) and Solow (1986), many practical measurements of sustainability assume that it does. But in fact it does not. There is a simple intuitive reason for this, already mentioned in Chapter 1. To calculate aggregate wealth, we use relative prices, adapted in some way from current market prices, to value the contribution that different physical assets in the economy make to wealth. A general, or even a PV-optimal development path is not subject to any sustainability constraint. So there is no reason why relative prices – in particular, the prices of natural resources relative to human-made capital – on such a path should tell us anything about sustainability. This is true even if the prices are adapted to include non-marketed environmental values, since (as Chapter 2 has shown at length), environmental correctness and sustainability (i.e., NDU) are different concepts.

More formally, I show below that the PV-optimal path of a single-peaked, initially sustainable economy is *bound* to go through a period when aggregate wealth is rising but the economy is unsustainable. To reach this result, I first need to recapitulate portions of Asheim (1994), and also of Weitzman's paper on the welfare interpretation of net national product.

Then in Section 3.2.2 I discuss what the national income accounting literature has to say about sustainability, and how the confusion referred to above has arisen. The extent of my contribution is reflected in the attribution of the six main propositions that are proved along the way. The first four propositions are quite general and apply to any economy with sufficient smoothness and convexity, not just to a single-peaked capital-resource economy.

3.2.1 Derivation of the main results

We assume an economy similar to that described by Dixit, Hammond and Hoel (1980), hereafter **DHH**, and Asheim (1994), with perfectly competitive firms and consumers, perfect information and complete markets, including all forward markets. Population is constant, consumers are identical and ageless, so they can be represented by a single, infinitely-lived consumer. She derives instantaneous utility $U(C)$ from scalar consumption C,[7] and seeks (so we assume) to maximise "PV", which we define as the present discounted value of utility *using a constant, positive utility discount rate* δ:[8]

$$\text{MAX} \quad \text{PV} := \int_0^\infty U[C(t)]e^{-\delta t}dt, \; \delta > 0 \qquad (3.1)$$

Let Σ be the vector of all stocks of human-made capital and natural resources in the economy, with an initial endowment $\Sigma(0) = \Sigma_0$. Feasible choices available at any time are all vectors $(C,\Sigma,\dot{\Sigma}) \in \Pi$, the smooth and convex production possibilities set. Note that Π has no time dependence.

7. DHH worked with multiple consumption goods and labour in the utility function, but we assume a fixed labour supply and this greater generality is not relevant here.

8. As is clear from DHH, p552, it is a basic *axiom* (though perhaps a questionable one) that consumers maximise present value. DHH and Asheim, often use a general discount factor $\lambda(s)$ rather $e^{-\delta t}$. But since we are specifically interested in the effects of the latter form, it is more convenient to define "PV" as we have done here.

As Weitzman (p157) says, this assumes that "*all* sources of economic growth have been identified and attributed to one or another form of capital, broadly defined", i.e. ***there is no exogenous technical progress***. Added to the other assumptions made in this section, this defines what we shall call *a Weitzman economy*, of which a Dasgupta-Heal economy is a special case.

We assume that growth in the economy is sufficiently bounded for a solution to the problem (3.1) to exist, called $\widetilde{C}(t)$, $\widetilde{\Sigma}(t)$, the *PV-optimal* paths of consumption and assets over time. Then by the maximum principle there also exist supporting *current* value prices $\widetilde{\psi}(t)$ and $\widetilde{\pi}(t)$ in terms of utility,[9] such that

(i) for each t, $\{\widetilde{C}(t),\widetilde{\Sigma}(t),\dot{\widetilde{\Sigma}}(t)\}$ maximizes instantaneous profit $\widetilde{\psi}C+\widetilde{\pi}.\dot{\Sigma}+(\dot{\widetilde{\pi}}-\delta\widetilde{\pi}).\Sigma$ subject to $(C,\Sigma,\dot{\Sigma}) \in \Pi$,

(ii) for each t, $\widetilde{C}(t)$ maximises $U(C)-\widetilde{\psi}C$ over all C.

We also assume that the PV-optimal path satisfies the transversality condition, so that the path is 'regular' and we may use Asheim's results:

(iii) $e^{-\delta t}\widetilde{\pi}(t).\widetilde{\Sigma}(t)\to 0$ as $t\to\infty$.

The *current PV* of any development path U is its PV from time t onwards, $\int_t^\infty U(s)e^{-\delta(s-t)}ds$, denoted $\mathrm{PV}_t(U)$. The *equivalent constant utility level* of the path is $\delta[\mathrm{PV}_t(U)]$, which is the *hypothetical* level of utility that, if held constant forever, *would* have a PV equal to $\mathrm{PV}_t(U)$. Contrast this to the *current maximin* utility U_t^m, defined above as the maximum constant utility level that an economy actually *can* sustain forever from time t to ∞, starting with the capital and resource stocks existing at t. Except at time zero, these stocks are path-dependent, so the current maximin is also path-dependent, and hence is denoted $\widetilde{U}_t^m$ on the PV-optimal path. We frequently

9. π is related to the *present* value utility prices q which Asheim uses by $\pi=e^{\delta t}q$.

use the well-known fact that the current maximin is Pareto-efficient, and also the distinction drawn in Chapters 1 and 2 between sustainedness over time ($\dot{U} \geq 0$ for all t) and sustainability at time t (the existence of a sustained ath starting from $U(t)$ and the capital and resource stocks existing at t). It is trivial to show that an economy is sustainable at t if and only if $U(t) \leq U_t^m$, and unsustainable at t if and only if $U(t) > U_t^m$.

Now define *net national welfare* Z on the PV-optimal path as

$$\widetilde{Z}(t) := \widetilde{U}(t) + \widetilde{\pi}(t).\dot{\widetilde{\Sigma}}(t) \tag{3.2}$$

where. From Asheim (1994, p260), in my notation:

$$d(e^{-\delta t}\widetilde{Z})/dt = -\delta \widetilde{U} e^{-\delta t}$$

$$\Rightarrow \qquad \dot{\widetilde{Z}} = \delta(\widetilde{Z} - \widetilde{U}) \tag{3.3}$$

$$\Rightarrow \qquad \dot{\widetilde{Z}} = \delta \widetilde{\pi}.\dot{\widetilde{\Sigma}} \tag{3.4}$$

$$\text{or} \Rightarrow \quad e^{-\delta t}\widetilde{Z}(t) - e^{-\delta T}\widetilde{Z}(T) = \delta \int_t^T \widetilde{U}(s) e^{-\delta s} ds. \tag{3.5}$$

Assume as above that the PV integral (3.1) converges, and also that $\lim_{t \to \infty} e^{-\delta t}\widetilde{\pi}.\dot{\widetilde{\Sigma}} = 0$ (i.e. Asheim's regularity conditions), so that $\lim_{T \to \infty} e^{-\delta T}\widetilde{Z}(T) = 0$. Now let $T \to \infty$ in (3.5) and manipulate to get

$$\widetilde{Z}(t)/\delta = \int_t^\infty \widetilde{U}(s) e^{-\delta(s-t)} ds = \mathrm{PV}_t(\widetilde{U}) \tag{3.6}$$

This is Weitzman's 'main proposition' in utility units, which we number as:

PROPOSITION 3.1 (Anon, after Weitzman): *In a Weitzman economy, $\widetilde{Z}(t)$, net national welfare on a PV-optimal utility path, equals $\delta[PV_t(\widetilde{U})]$, the equivalent constant utility level of the path.*

This Proposition does *not* say that utility level $\widetilde{Z}$ is *feasible* forever, for an important corollary (noted in passing by Asheim's Section III, but deserving greater prominence in my view) is:

PROPOSITION 3.2 (Asheim/Pezzey): *(i) On the PV-optimal path of any Weitzman economy, current maximin utility* $\widetilde{U}^m_t$ *is less than or equal to net national welfare* $\widetilde{Z}(t)$. *(ii) In almost all such economies,* $\widetilde{U}^m_t < \widetilde{Z}(t)$.

PROOF:

(i) Assume to the contrary that $\widetilde{U}^m_t > \widetilde{Z}(t)$. Then by Proposition 3.1, $PV_t(\widetilde{U}) = \widetilde{U}^m_t/\delta > PV_t(\widetilde{U})$. But this contradicts the PV-optimality of $\widetilde{U}$. ||

(ii) The PV-optimal utility path is unique, and for 'almost all' economies it is *not* constant over time. (That is, although one can think of a one-sector, non-resource economy which PV-optimally starts and remains in a steady state with constant utility, such a special case is of no interest here. Throughout the rest of Section 3.2, this will be the precise meaning of the phrases *almost all* or *almost always*.) So any feasible constant utility path such as the current maximin, has strictly lower PV than the PV-optimal path. That is, $\widetilde{U}^m_t/\delta < \mathrm{PV}_t(\widetilde{U})$. But $\mathrm{PV}_t(\widetilde{U}) = \widetilde{Z}(t)/\delta$, from Proposition 3.1, so $\widetilde{U}^m_t < \widetilde{Z}(t)$. ||

Next, note that $\pi.\dot{\Sigma}$ represents a measurement in utility units of *aggregate investment*, i.e. investment in reproducible capital minus the sum of resource rentals in the absence of extraction costs. If $\pi.\dot{\Sigma}=0$ (Hartwick's Rule in utility units), then consumption (and hence utility) is constant, $\dot{C}=\dot{U}=0$ (Hamilton 1994). The time integral of aggregate investment will be called *aggregate wealth* Ψ:

$$\Psi(t) := [Z(0)/\delta] + \int_0^t \pi(s).\dot{\Sigma}(s)ds. \tag{3.7}$$

$\pi.\dot{\Sigma} < 0$ is thus *declining aggregate wealth measured at prices* $\wp$. Aggregate wealth is a backward-looking measure of what the economy has accumulated, in contrast to net national welfare divided by δ, which is an instantaneous measure of wealth, and PV, which is a forward-looking measure of what the economy will enjoy between t and ∞. Then from (3.4) and (3.7):

$$\dot{\widetilde{Z}} = \delta\dot{\widetilde{\Psi}} \tag{3.8}$$[10]

so that the increment in net national welfare on a PV-optimal path can be seen as the 'interest' (at rate δ) on the increment in aggregate wealth $\widetilde{\Psi}$. Since both $\widetilde{Z}$ and $\delta\widetilde{\Psi}$ are *continuous* (the former follows from an extension of Lemma 3.5 in Appendix 3.3), have the same value at $t=0$ by (3.7), and have the same derivative for all $t \geq 0$ by (3.8), they are equal:

PROPOSITION 3.3 (Pezzey, after Solow): *Net national welfare $\widetilde{Z}$ on the PV-optimal path of a Weitzman economy equals the utility discount rate δ times aggregate wealth $\widetilde{\Psi}$.*

Another simple but important result regarding aggregate wealth is:

PROPOSITION 3.4 (Pezzey, after Solow and Pearce): *In a Weitzman economy, declining aggregate wealth on the PV-optimal path ($\widetilde{\wp}.\dot{\widetilde{\Sigma}} < 0$) implies unsustainability ($\widetilde{U}^m_t < \widetilde{U}(t)$).*

PROOF:

From (3.2), $\widetilde{\pi}(t).\dot{\widetilde{\Sigma}}(t) < 0 \;\Rightarrow\; \widetilde{Z}(t) < \widetilde{U}(t)$. Together with Proposition 3.2(i), this gives $\widetilde{U}^m_t < \widetilde{U}(t)$. ||

Of course, if an economy is not on its PV-optimal path, net national welfare (defined simply as $U + \pi.\dot{\Sigma}$) could be below $\widetilde{Z}$, and so could be the same as U^m_t, but this would be a mere coincidence.

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10. Another measure of wealth is the current utility value of all stocks in the economy. Call this *gross wealth* and denote it by $\Omega := \pi.\Sigma$. Then $\dot{\Omega} = \pi.\dot{\Sigma} + \dot{\pi}.\Sigma$, so that within an additive constant, Ω equals aggregate wealth plus stock appreciation (the time integral of $\dot{\pi}.\Sigma$). But by (3.4) and (3.6), Ω could be rising while the equivalent constant utility level δPV is falling, which is why we focus on aggregate wealth Ψ instead.

Next, one can find a counterexample where *rising* aggregate wealth coincides with unsustainability. Asheim (1994) claimed to have found one, but as noted in Section 3.3 below, his argument was incomplete because it used an unproved assertion borrowed from Dasgupta and Heal (1979), hereafter **DH79**. In Appendix 3.2 I do find a counterexample, using the Special Case of Section 3.3, but in fact a more striking, general result can be shown:

PROPOSITION 3.5 (Pezzey, after Asheim): *In any Weitzman economy where the PV-optimal path is single-peaked and initially sustainable, there will be a finite time period during which the economy is unsustainable but aggregate wealth is rising.*

PROOF:

We use a slightly informal graphical proof to make the basic reasoning clear. We assumed a smooth and convex production function, so the PV-optimal utility, net national welfare and current maximin curves will be smooth and continuous. **Figure 3.2** shows the single-peaked PV-optimal utility path $\widetilde{U}$ with a peak at time T_P. The assumed initial sustainability means that $U_0^m > \widetilde{U}(0)$ as shown, and Proposition 3.2 $\Rightarrow$ $\widetilde{Z}(0) > U_0^m$, as shown.

Since the equivalent constant level of a declining path must always be less than the current value on the path, by Proposition 3.1, $\widetilde{Z} < \widetilde{U}$ after T_P, as shown. By continuity of $\widetilde{Z}$ between 0 and T_P, there is at least one time T_H (say 'Hartwick's time') with $0 < T_H < T_P$ when $\widetilde{Z} = \widetilde{U}$ and hence $\widetilde{\pi}.\dot{\widetilde{\Sigma}} = 0$ (aggregate investment is zero); and it is unique since by (3.3), $\dot{\widetilde{Z}} = \delta(\widetilde{Z} - \widetilde{U}) < 0$ between T_H and T_P, but $\dot{\widetilde{U}} > 0$ then. Also, from $\dot{\widetilde{Z}} = \delta(\widetilde{Z} - \widetilde{U})$ it can readily be proved that $\widetilde{Z}$ is single-peaked with its maximum at T_H, as shown.

Likewise, at $t=0$ the current utility maximin $\widetilde{U}^m > \widetilde{U}$ by assumption, and after $t=T_P$, $\widetilde{U}^m < \widetilde{Z} < \widetilde{U}$ (by Proposition 3.2 and the start of this paragraph). So by continuity there is a time T_L between 0 and T_P when $\widetilde{U}^m = \widetilde{U}$. T_L is also unique, since $\widetilde{U}^m < \widetilde{U} \Rightarrow \dot{\widetilde{U}}^m \leq 0$. (For if $\dot{\widetilde{U}}^m > 0$, one could strictly dominate $\widetilde{U}^m$ starting from any given time t by following $\widetilde{U}$ instead for a short while till some $t+\epsilon$, and then dropping down to the constant utility level $\widetilde{U}^m(t+\epsilon) > \widetilde{U}^m(t)$. But $\widetilde{U}^m(t)$, being a maximin level and therefore efficient, cannot be so dominated, giving a contradiction.) By Proposition 3.2 and the continuity of $\widetilde{U}$, $T_L < T_H$. So during the finite time interval $T_L < t < T_H$, $\widetilde{U}^m < \widetilde{U}$ (the PV-optimal path is unsustainable) but $\widetilde{Z} > \widetilde{U} \Rightarrow \widetilde{\pi}.\dot{\widetilde{\Sigma}} > 0 \Rightarrow \dot{\widetilde{\Psi}} > 0$ (aggregate wealth is rising). ||

Intuitively, and as already noted in Chapter 1, rising aggregate wealth coincides with unsustainability in the above example because wealth is being measured at 'unsustainable' prices. On the PV-optimal path, non-renewable resources are being used up too rapidly for sustainability, which means that their prices must be lower. This reduces the negative (resource depletion) components of aggregate investment $\pi.\dot{\Sigma}$, yielding a 'false positive' value, which conceals the fact that the true 'sustainability' cost of resource depletion is more than the value of capital investment at that time.

Finally, we can use Proposition 3.5 to calculate the effect of an 'unanticipated constant utility policy at T_H', i.e. a set of unanticipated policy incentives which start at T_H and somehow achieve $\pi.\dot{\Sigma}=0$ (and hence $\dot{U}=0$) from T_H onwards. (We drop the tildes here because constant utility is not PV-optimal.) We then have:

PROPOSITION 3.6: *If PV-optimal consumption in a Weitzman economy is single-peaked and initially has rising aggregate wealth, and the economy follows the PV-optimal path until an unanticipated constant utility policy is introduced at T_H when aggregate wealth is at a maximum, then*

$t<T_H \;\Rightarrow\; U^m_t,\ U(t) \;<\; Z(t) \;=\; \delta\Psi(t) \;>\; \delta PV_t(U);$

$t=T_H$: *$U(t)$ and $Z(t)$ are discontinuous downwards, while U^m_t, $\delta\Psi(t)$ and $\delta PV(t)$ are continuous;*

$t>T_H \;\Rightarrow\; U^m_t \;=\; U(t) \;=\; Z(t) \;=\; \delta PV_t(U) \;<\; \delta\Psi(t).$

PROOF:

Since the policy is unanticipated, all current or backward-looking measures on the economy's actual path take PV-optimal values before T_H, as labelled in **Figure 3.3**. Hence U^m_t, $U(t) < Z(t) = \delta\Psi(t)$ for $t<T_H$ from the PV-optimal results above. But the forward-looking measure $\mathrm{PV}_t(U)$ (not shown on the Figure) is less than aggregate wealth Ψ, because U is not PV-optimal after T_H.

At T_H, the economy jumps to what is labelled the 'policy path', in response to the constant utility policy. As marked by the small arrows, utility U jumps downwards from the PV-optimal level at H to the current maximin at J. Since $\pi.\dot{\Sigma}=0$ both just before and just after T_H, net national welfare $Z=U+\pi.\dot{\Sigma}=U$ before and after T_H, so Z also jumps down from H to J at T_H. In contrast, aggregate wealth Ψ, being the integral of $\pi.\dot{\Sigma}$, is continuous. $\delta\Psi$ therefore remains on a level with H, giving $\delta\Psi > Z$ after T_H. By the assumption that utility is at the maximum sustainable level after T_H, actual utility U ($=Z$) and current maximin utility U^m are equal to each other then, and also to $\delta[\mathrm{PV}(U)]$ (not shown on Figure 3.3). ||

3.2.2 *Comparison of the above results with the existing literature*

In comparing the above results with the literature on national income accounting, we face the immediate problem that Weitzman (1976), Solow (1986) and many other authors assume that the economy acts as if to maximise $\int_t^{\infty} C(t)e^{-rt}dt$, the present value of *consumption* (rather than utility) using a constant *interest* rate r (rather than utility discount rate δ). But on a PV-optimal development path, $\tilde{r} = \delta - \dot{\tilde{C}}\tilde{U}_{CC}/\tilde{U}_C = \delta + (\dot{\tilde{C}}/\tilde{C})\eta(\tilde{C})$ (DH79, p293). That is, the PV-optimal interest rate is constant only when the PV-optimal growth rate of consumption is constant and the utility function is isoelastic, or when the marginal utility of consumption is constant ($\Rightarrow U_{CC}=\eta=0$). When we are specifically concerned with rising and then falling consumption, the first circumstance is excluded, so we have to assume $U_{CC}=0$. In this case it is not at all certain that a well-defined PV-optimal path exists! However, this is a standard problem that I leave to investigation by others; in any case, changes in net national product do have the correct sign as the change in welfare.

If $U_{CC}=0$ (i.e. utility is a linear function of consumption, say without loss of generality that $U(C)=C$ and $U_C=1$), then many of the above variables in the economy can take on a different name, without changing the validity of the above results:

- utility-based prices $\boldsymbol{\pi}$ become consumption-based prices, say $\boldsymbol{p}$;
- net national welfare Z becomes net national product (NNP) Y, now defined as $C+\boldsymbol{p}.\dot{\boldsymbol{\Sigma}}$;
- aggregate wealth Ψ becomes what Solow (1986) defines as $V(t)=[Y(0)/\delta]+\int_0^t \boldsymbol{p}.\dot{\boldsymbol{\Sigma}}$, called 'aggregate consumption wealth' here;
- PV becomes PVC, the present value of consumption;
- the utility discount rate δ also becomes the interest rate r.

The results then become:

If the marginal utility of consumption is constant and the PVC-optimal path exists:

PROPOSITION 3.1′: *In a Weitzman economy, $\widetilde{Y}(t)$, NNP on a PVC-optimal path, equals $r[PVC_t(\widetilde{C})]$, the equivalent constant consumption level of the path.*

PROPOSITION 3.2′: *(i) On the PVC-optimal path of a Weitzman economy, current maximin consumption $\widetilde{C}^m_t \leq$ NNP $\widetilde{Y}(t)$. (ii) In almost all*[11] *such economies, $\widetilde{C}^m_t < \widetilde{Y}(t)$.*

PROPOSITION 3.3′: *NNP $\widetilde{Y}$ on the PVC-optimal path of a Weitzman economy equals the interest rate r times aggregate consumption wealth $\widetilde{V}$.*

PROPOSITION 3.4′: *For all Weitzman economies, declining aggregate consumption wealth on the PVC-optimal path ($\widetilde{p}.\dot{\widetilde{\Sigma}} < 0$) implies unsustainability ($\widetilde{C}^m_t < \widetilde{C}(t)$).*

PROPOSITION 3.5′: *In any Weitzman economy where the PVC-optimal path is single-peaked and initially sustainable, there will be a finite time period during which the economy is unsustainable but aggregate consumption wealth is rising.*

PROPOSITION 3.6′: *If PVC-optimal consumption in a Weitzman economy is single-peaked and initially has rising aggregate wealth, and the economy follows the PVC-optimal path until an unanticipated constant utility policy is introduced at T_H when aggregate wealth is at a maximum, then*

11. Recall that the meaning of 'almost all' was defined in the proof of Proposition 3.2(ii).

$$t<T_H \Rightarrow C^m_t,\ C(t) < Y(t) = rV(t) > rPVC_t(C);$$

$t=T_H$: *C(t) and Y(t) are discontinuous downwards, while C^m_t, rV(t) and rPVC(t) are continuous;*

$$t>T_H \Rightarrow C^m_t = C(t) = Y(t) = rPVC_t(C) < rV(t).$$

To compare the above, consumption- rather than utility-based results with the national income accounting literature, it also helps to have a clear, undisputed definition of *income*. But the author of perhaps the most famous definition (which we will analyse shortly) warned us that this may not be easy:

> "We have seen eminent authorities confusing each other and even themselves, by adopting different definitions of saving and income, none quite consistent, none quite satisfactory. When this sort of thing happens, there is usually some reason for the confusion; and that reason needs to be brought out before any further progress can be made." (Hicks 1946, p171)

A recent explanation of this confusion is that a single definition is inherently impossible, because national income serves many *different purposes*, such as:

> "...charting business cycles, comparing prosperity among nations, observing industrial structure, measuring factor shares and so on. ...real income may be interpreted as a family of concepts, each member of which is best for some particular purpose." (Usher 1994, p124)

These different purposes have given rise to a number of controversies, such as on the use of net versus gross measures of national product in accounting for economic growth.[12] Hulten (1992, p9) argues that "gross product is the correct output concept for estimating the structure of production, while net product is the correct concept for measuring the

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12. The difference between net and gross product is basically the amount of capital depreciation. The difference between income and product is basically the income from net foreign assets, which will not exist here because the economy is closed.

welfare consequences of economic growth". However, this is not a thesis on national income accounting as such, so I shall ignore this issue and study only parts of the accounting literature that touch on sustainability. I start with two concepts of income in the context of constant interest rates:

I1: *maximum sustainable income*, defined as the maximum scalar C such that 'consumption $C(t)=C$ for all t' is feasible, but 'consumption $C(t)=C+\epsilon$ for all t' is infeasible for all $\epsilon>0$; that is, the current maximin consumption C_t^m, as defined earlier.

I2: *income as consumption plus aggregate investment on a PVC-optimal path*, i.e. the optimal NNP $\widetilde{Y}(t)$, by our earlier definition. Recall that by Proposition 3.1′ (the main result in Weitzman 1976), this is also the interest rate r multiplied by maximum wealth, where in turn wealth is defined as present discounted value of future consumption.

Even though Hicks ends up advising us (p177), "...to eschew *income* and *saving* in economic dynamics. They are bad tools, which break in our hands." – definitions I1 and I2 will serve to clarify a good deal of confusion.

For one thing, they are clearly *different* concepts. By Proposition 3.2′(ii), maximum sustainable income is almost always strictly less than consumption plus aggregate investment on a PVC-optimal path. However, I will now argue, using lengthy quotations which should enable the reader to form his own opinion, that the two concepts have become badly confused in the literature. Some confusion is purely semantic, and arises because someone chooses to define NNP as maximum sustainable income rather than as PVC-optimal consumption-plus-aggregate-investment, while acknowledging that the latter two are different.[13] But more commonly, some authors seem to think that NNP, maximum sustainable income and PVC-optimal consumption-plus-aggregate-investment *are always equivalent*, which is incorrect.

My first quotation is the famous one, which it is important to give in full:

> "The purpose of income calculations in practical affairs is to give people an indication of the amount which they can consume without impoverishing themselves. Following out this idea, it would seem that we ought to define a man's income as the maximum value which he can consume during a week, and still expect to be as well off at the end of the week as he was at the beginning. Thus, when a person saves, he plans to be better off in the future; when he lives beyond his income, he plans to be worse off. Remembering that the practical purpose of income is to serve as a guide for prudent conduct, I think it is fairly clear that this is what the central meaning must be." (Hicks 1946, p172)

Although Hicks then goes on to show (p176) "how very complex [this central meaning] is, how unattractive it looks when subjected to detailed analysis", in considering how changes in the interest rate can affect this definition of income, he also offers (p174):

> "...definition of Income No. 2. We now define income as the maximum amount the individual can spend this week, *and still expect to be able to spend the same amount in each ensuing week.*" [italics added]

The italicised phrase here gives a much clearer idea of what Hicks meant on his p172 by being "as well off at the end of the week", at least in the simplified world of a Weitzman economy. It surely means that we should equate Hicks' central meaning with definition I1, i.e. maximum sustainable income, and *not* with definition I2, the interest on maximum wealth. And why else would Hicks be concerned with 'prudent', rather than 'optimal',

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13. An example is Asheim (1994, p257, emphasis added) who argued that "NNP *should* equal the maximum per capita consumption level that can be sustained", although he then went on to show that the consumption-plus-investment definition of NNP *cannot* (except in trivial cases) equal this.

conduct on his p172? Why else would he be concerned with avoiding future 'impoverishment', which DH79 (p299) showed can well occur on a wealth-maximising path? Scott (1990, p1173), Eisner (1990, p1180, who also quotes Hicks from p174) and Asheim (1994, p257) would all agree with this formal interpretation of Hicksian income.

Historically, the confusion I allude to seems to have started from the following passage contained in, but not central to, Weitzman's classic paper on the welfare interpretation of NNP:

> "Even granted that consumption is the ultimate end of economic activity, the national income statistician's practice of adding in investment goods to the value by weighting them with prices measuring their marginal rates of transformation might still be defended as a measure of the economy's power to consume at a constant rate. After all, a standard welfare interpretation of NNP is that it is the largest permanently maintainable value of consumption. If all investment were convertible into consumption at the given price-transformation rates, the maximum attainable level of consumption that could be maintained forever without running down capital stocks would appear to be NNP as conventionally measured by $\widetilde{C}+\widetilde{p}.\dot{\widetilde{\Sigma}}$." (Weitzman 1976, p159, using my notation)

This passage is arguably inconsistent. For Weitzman explicitly noted (p159) that NNP defined as $\widetilde{C}+\widetilde{\boldsymbol{p}}.\dot{\widetilde{\Sigma}}$ is "just the Hamiltonian for a general [*PVC*]-optimization problem". He mentioned no extra conditions which would make the economy's PV-optimal time path of consumption constant in general. Proposition 3.2′ therefore holds, and maximum sustainable income is almost always less than NNP. So ***if*** one interprets Weitzman's "the largest permanently maintainable value of consumption" and "the maximum attainable level of consumption that could be maintained forever" as maximum sustainable income (definition I1 above) – and this is surely a natural interpretation – then neither of these concepts can be generally interpreted as NNP, contrary to what he said. True, he went on to doubt in his next paragraph that maximum sustainable income would be less than NNP if investment is not "convertible into consumption at the given price-transformation rates". But even if investment *is* thus convertible, NNP (definition I2) would *still* not generally equal maximum sustainable income (definition I1).[14]

Maximum sustainable income certainly was a central issue of debate for Solow (1986), and even more so for the many people who have cited him. However, it is easier to see his complete argument by quoting a later paper:

> "Something interesting happens when these two propositions are put together. One of them tells us that NNP at any instant is a measure of the highest sustainable income achievable, given the stock of capital available at that instant. The other proposition tells us that NNP at any instant can be represented as that same stock of capital multiplied by an unchanging discount rate. Suppose that one goal of economic policy is to make investment and depletion decisions this year in a way that does not erode sustainable income. Then those same decisions must not allow the aggregate capital stock to fall. To use a Victorian phrase, preserving sustainability amounts to maintaining society's capital intact." (Solow 1993, p169)

(Solow's 'aggregate capital' is the same, albeit in consumption rather than utility units, as my 'aggregate wealth'.) There is a nice resourcist flavour to the "stock of capital" argument here, which bears further thought, as noted in Chapter 1. Perhaps we should care only whether or not the *opportunity* for sustainable consumption is declining, not whether consumption (and hence utility) actually *is* declining. In either case, to interpret the full quotation we first need to know what Solow meant by NNP. The following shows that he does mean consumption-plus-aggregate-investment, evaluated at the 'right prices':

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14. Two further points arise here. First, precisely this convertibility assumption is made by the common linear production function $F(K,R)=\dot{K}+C$ used in scores of growth models (e.g. by DH74, p9). Second, the *instantaneous* production possibility frontier in $(\dot{K},C)$-space illustrated by Weitzman (p160) is not well-defined in a resource economy, because the *instantaneous* resource flow rate R, which is one factor of production, remains undetermined. But these are side issues.

"...properly defined net product, calculated with the aid of the right prices. ... The economy's net product in any year consists of public and private consumption and public and private investment. ... The components of investment, including the depletion of natural resources, have to be valued. That is where the 'rightness' of the prices comes in. ... The right prices will make full allowance even for the distant future, and will even take account of how each future generation will look at its future." (Solow 1993, pp168-9).

But *it is not clear whether Solow's 'right prices' were the prices on the maximum sustainable income path (call them p^m), or the prices on the PVC-optimal path ($\tilde{p}$)* – or even whether Solow recognised that the two sets of prices are almost always different (which they are, by Proposition 3.2′). This is the ultimate source of the confusion arising from Solow's papers. *If* he meant that PVC-optimal prices are 'right', and *if* we make a natural interpretation of his 'highest sustainable income achievable' as maximum sustainable income (definition I1), then the first quotation would be incorrect: NNP would not be the highest sustainable income, and maintaining intact society's capital *as measured at PVC-optimal prices* will not preserve sustainability.

If, on the other hand, by 'right prices' Solow meant 'sustainable' prices (p^m), then his argument would hold. However, *there is no way of measuring sustainable prices from the national income accounts, unless the government has already introduced an explicit policy to preserve sustainability*; this was the main conclusion of Asheim (1994). As we shall see later in the Chapter, this does not necessarily mean that some 'sustainability tax' must already be in effect, but it does mean that the government must have made a credible announcement that such a tax will be introduced when it is needed. Solow did not seem to recognise this. On p146 of his 1986 paper, he defined his prices as the shadow prices on a PVC-optimal path (i.e. as $\tilde{p}(t)$ in my notation). But then on p147 he said (in my notation):

"Now suppose that $p(t).\dot{\Sigma}(t) = 0$ from some date on. This is Hartwick's rule."

Let us call this date T. Three things must happen at T, none of which Solow mentioned: (1) the government policy is credibly announced; (2) prices respond by jumping from $\tilde{p}(T-)$, the PVC-optimal prices holding just before T, to $p^m(T+)$, the 'sustainability' prices holding just after T; (3) NNP jumps from $\tilde{Y}(T-)$ to $\tilde{Y}^m(T+)$,[15] and hence there is a 'spike' in its derivative $\dot{Y}$ at T.

This last explains the technical flaw in Solow's 1986 result. His contribution on pp147-8 was to point out that Weitzman's equation $\dot{\tilde{Y}}=r(\tilde{Y}-\tilde{C})$ (equation (3.3) translated into consumption units) could be integrated in a different way. Weitzman integrated this forward from t to ∞ to produce

$$\tilde{Y}(t) = r\int_t^\infty \tilde{C}(s)e^{-\delta(s-t)}ds,$$

the interpretation we have already seen of NNP as the interest on the present value of future consumption. Solow instead integrated $\dot{\tilde{Y}}=r\tilde{p}.\dot{\Sigma}$ (the consumption-unit version of (3.4)) from 0 to t to give

$$\tilde{Y}(t) = \tilde{Y}(0) + r\int_0^t \tilde{p}(s).\dot{\Sigma}(s)ds = \tilde{Y}(0) + r\tilde{V}(t)$$

which allows

> "the increment in NNP since $t=0$ [to be] representable as interest on the accumulation of capital value since $t=0$, in an inclusive sense that records the decumulation of the stock of exhaustible resources." (Solow 1986, p147)

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15. An example of this jump calculated for a specific functional form can be seen on Figure 3.3, albeit in utility rather than consumption units. Time T_H, when an unanticipated constant utility policy is introduced, happens to be when aggregate wealth measured at PV-optimal prices is at a maximum, but there would be a downward jump in net national welfare for any other such time T.

This is fine as long as we are on the PVC-optimal path, but *not* once time t extends after T, as Solow clearly intends it should do. For although $\dot{Y}=r\boldsymbol{p}.\dot{\Sigma}$ is still true after T when Hartwick's rule holds (for both sides of the equation are zero, since aggregate investment is zero and consumption is constant), simply integrating this equation past T ignores the spike in $\dot{Y}$ (i.e. the discontinuity in Y) caused by the policy announcement. So although *some* measure of aggregate wealth is constant after T, because aggregate investment $\boldsymbol{p}.\dot{\Sigma}$ is zero, it is not aggregate wealth *measured at PVC-optimal prices*. Hence non-declining aggregate wealth at PVC-optimal prices does not guarantee sustainability.

Does this confusion about NNP, maximum sustainable income and the 'right prices' to use in measuring aggregate wealth, that I have shown exists in Weitzman (1976) and Solow (1986) – both famous papers, each with 20-30 citations during 1981-94 logged by SSCI – actually matter? Have people been misled by it? And what will happen if people are no longer misled?

It does matter, because measuring aggregate wealth has much less predictive power for sustainability than many people have assumed. Proposition 3.4/3.4′ means that falling aggregate wealth, measured at PVC-optimal prices, does imply unsustainability – but also that it is already too late to stop it. Conversely, finding that aggregate wealth is rising may give a false positive message, as noted before. We may be reassured about sustainability because the PV-optimal resource price is 5 and $\dot{\widetilde{K}}-5\widetilde{R}>0$, when in fact we should be alarmed because the 'sustainability' price of the resource is actually 7, and $\dot{\widetilde{K}}-7\widetilde{R}<0$.

And people have been misled or confused. The quotations below, followed by my comments, show this. (Asheim 1994 noted the claims by

Maler and Hulten, but not the others.) Some of the papers quoted are mainly concerned with issues other than sustainability, such as how correctly to reflect pollution damage and clean-up expenditures in the national accounts, but we will focus solely on what the papers say about NNP and sustainability. We start with quotations from Pearce and his school, who are very influential, and have made valiant efforts in the heroic task of measuring sustainability and influencing policy with these measurements.

> "Hartwick [1977] showed that a society with an exhaustible resource, such as oil, could enjoy a constant stream of consumption over time provided it invested all the 'rents' from the exhaustible resource. (A 'rent' is the difference between the price obtained for the resource and its costs of extraction.) What Solow [1986] shows is that the Hartwick rule is formally equivalent to *holding the overall capital stock constant*. The constant stream of consumption is then viewed as the interest secured on that 'patrimony'." (Pearce, Markandya and Barbier 1989, p50; italics in original).

> "...reinvestment of the total 'rent' from exhaustible resource exploitation will secure a constant stream of consumption over time, which is thus 'intergenerationally fair'. Solow (1986) demonstrates that such a fairness rule is formally equivalent to keeping the stock of all capital constant." (Barbier, Markandya and Pearce 1990, p1260).

> "Unless there are special reasons for singling out one form of capital, the requirement for sustainable development then becomes one of passing on to the next generation an aggregate capital stock no less than the one that exists now (Hartwick, 1978; Solow, 1986)." (Pearce et al 1993, p15)

> "...we adopt a neoclassical stance and *assume* the possibility of substitution between 'natural' and 'man-made' capital....in the sense described by Victor (see Solow 1986). We then assert that an economy is sustainable if it saves more than the *combined* depreciation on the two forms of capital." (Pearce and Atkinson 1993, p104)

The above quotations show that Solow's lack of clarity led Pearce et al to ignore altogether the importance of using sustainability prices to measure rents and thus 'overall capital', 'all capital' or 'aggregate capital' (that is, aggregate wealth). Even though they are well aware that "Valuation problems, especially with functions such as contributions to reducing future [environmental] catastrophes, are formidable" (Pearce, Markandya and Barbier 1989, p44), they do not recognise that PV-optimal prices and sustainability prices are different, even when there is no critical positive level of the resource below which catastrophe occurs.

Another much-cited paper is by Maler (1991), who was mainly concerned with the theory of how environmental damages and clean-up expenditures should be reflected in the national accounts. But in so doing he states:

> "The present value of the constant utility stream H^* is thus equal to the maximum present value of the utility stream. Thus $H^*(t)$ is the maximum current utility that can be sustained forever, that is, H^* (or NWM$=H^*$) is a measure of sustainable income (in utility terms)." (Maler 1991, p11)

On p5 Maler defines H^* as the Hamiltonian of an optimal control problem, which is Weitzman's definition of NNP $\widetilde{Y}$. Whether this is a PV-optimal or PVC-optimal problem is not clear, since although Maler claims to work in utility units, he uses a constant *interest* rate to discount utility, and nowhere allows for diminishing marginal utility. Whichever it is, *if* we make the natural interpretation that his "maximum current utility that can be sustained forever" is the utility derived from maximum sustainable income (definition I1), then Maler is almost always incorrect: if H^* could be *attained* as a constant utility path, then this path would have the same PV as the (time-varying) PV-optimal path, contradicting the latter's uniqueness.

Less influential papers with similar confusions are:

> "Solow (1986)...[has shown] that an intertemporal society that invests in reproducible capital the competitive rents on its current extraction of exhaustible resources, will enjoy a constant consumption stream" (Spash and d'Arge 1989, p92) [Almost always not true: competitive, that is, PV-optimal rents may be too low for sustainability.]

"The magnitude $\widetilde{C}(t)+\widetilde{p}(t).\dot{\widetilde{\Sigma}}(t)$ can be interpreted as the maximum amount of output that could be consumed without reducing the original amount of capital, or maximum sustainable consumption." (Hulten 1992, pS17, in my notation) [Almost always not true, *if* "maximum sustainable consumption" is interpreted as my maximum sustainable income (definition I1).]

"Weitzman (1976) defined the NNP as the 'largest permanently maintainable value of consumption'. This concept amounts to the Hicksian social income that can be represented by the current value Hamiltonian function of a dynamic economy." (Hung 1993, p380) [Almost always not true, *if* "largest permanently maintainable value of consumption" is interpreted as maximum sustainable income.]

"Maler (1991) showed that Proper Net Domestic Product as a measure of sustainable income can be defined and measured using the shadow, or imputed, prices that emerge from a dynamic optimization problem." (Common, Blamey and Norton, 1994) [Almost always not true, *if* there is no sustainability constraint applied to the dynamic optimization problem, and "sustainable income" is interpreted as maximum sustainable income.]

In answer to my third question, it is hard to predict what will happen if people no longer confuse NNP and maximum sustainable income, and come to realise that non-declining aggregate wealth does not guarantee sustainability. Theoretical research may begin to consider price differences between sustainable and unsustainable constant utility paths. Practical applications of my results will be harder, because of the uncertain technical progress which occurs in reality but has been ignored above (although it is also ignored in large swathes of the national accounting literature!). Perhaps the greatest impact of the above results will be philosophical, simply because people will be reminded that even perfected market forces cannot guarantee the long run future of civilisation.

3.3 SINGLE-PEAKEDNESS AND INITIAL SUSTAINABILITY OF RESOURCE ECONOMIES

Propositions 3.5-3.6 and Figures 3.1-3.2 assumed, without proof, the existence of a PV-optimal path of a capital-resource economy which is single-peaked (that is, with first rising and then falling utility) and initially sustainable. Here I give some moderately general conditions under which PV-optimal utility is either single-peaked or always falling (in which case it is unsustainable[16]); and then a much more restrictive case where PV-optimal utility is initially sustainable, and therefore single-peaked. Then in Section 3.3.2 I discuss the physical realism of these conditions.

One might be surprised that this exercise is necessary at all, since the properties are quite intuitive, and were mentioned long ago by DH74 (p17) and DH79 (p299). However, the existence of single-peakedness, which was claimed in DH79 for a Cobb-Douglas economy with diminishing returns to scale, was not actually proved there;[17] nor was it in the constant returns case in DH74.[18] And DH79's Diagram 10.3 (p299) implied that initial sustainability will occur for a low enough utility discount rate δ, and that initial unsustainability will occur for a high enough δ, but no proof was given. This lack of proof matters for the above propositions and for the properties of the opsustimal path to be derived in the next section. It also matters for Asheim (1994, p262). He relied on the results in Diagram 10.3 to deduce "by a continuity argument" that there is a δ (in my notation)

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16. By Lemma 3.2, if $\widetilde{U}$ initially falls it stays falling, which would then give it less PV than the initial maximin if $U_0^m \geq \widetilde{U}(0)$, contradicting the PV-optimality of $\widetilde{U}$.

17. Their analysis of the diminishing returns case ends (p302) with a pair of non-autonomous differential equations for resource flow and the capital stock, which are (indeed) "difficult to dissect in detail".

18. Single-peakedness does follow from their equation (1.37); but their assertion (p17) that it "would appear" to follow from having a *large* enough initial stock of capital was unproven, and moreover incorrect in the Special Case analysed below.

which makes PV-optimal utility and current maximin utility initially the same; and then used this δ to prove a weaker version of Proposition 3.5, namely that rising aggregate wealth does not necessarily imply sustainability.

3.3.1 Conditions for single-peaked or initially falling paths, and for initial sustainability

The most general conditions I know for single-peaked or initially falling PV-optimal paths to occur as are follows. The economy is the 'Dasgupta-Heal' economy of DH74 with just one capital stock and one resource stock, so it is already much less general than the economy of (3.1) which allowed for multiple capital and resource stocks. The PV-optimal utility path $\widetilde{U}(t)$ is the solution to

$$\underset{\{C(t),R(t)\}}{\text{MAX}} \quad \int_0^{\infty} U[C(t)]e^{-\delta t}dt \tag{3.9}$$

$$\text{s.t.} \quad \dot{K} = F(K,R) - C; \quad \dot{S} = -R \tag{3.10}$$

where U = utility and C = consumption as before, and now also F = output, K = capital, R = non-renewable resource depletion rate, and S = resource stock. Restrictions on these variables are:

$$K(0) = K_0 > 0, \;\; S(0) = S_0 > 0; \;\; C, K, R, S \geq 0; \tag{3.11}$$

U is twice continuously differentiable, with

$$U_C > 0 \text{ and } U_{CC} < 0; \tag{3.12}$$

$F(K,R)$ is twice continuously differentiable, non-negative, has a strictly positive elasticity of substitution between capital K and resource flow R, and is linearly homogeneous, so we can write

$[F(K,R)]/R = F(K/R,1) =: f(x)$ where $x := K/R$; (3.13)

Also $f(0)=0$ and $f(x)>0$ for $x>0$; (3.14)

and $F_K(K,R)=f'(x)>0$, $f''(x)<0$ for $x\geq 0$; (3.15)[19]

$0 \leq \lim|_{x\to\infty} f'(x) =: \rho < \delta$; (3.16)

$F_R \to \infty$ as $R \to 0$; (3.17)

$\sigma(x) \geq \bar{\sigma} > 0$, where $\sigma(x)$ is the elasticity of substitution between capital and resource flow. (3.18)

Note that:

- (3.10) allows capital consumption ($\dot{K}<0$), which is a key feature of many unsustained solutions to the above problem;
- the production function (3.13) omits labour as an input, and yet has constant returns to scale in the remaining inputs (capital and resources);
- in (3.16) the utility discount rate exceeds the limit of the marginal productivity of capital.

This last assumption is what drives PV-optimal consumption to zero asymptotically, thus creating a conflict between PV-optimality and sustainedness. We now prove the single-peakedness result for the solution to (3.9)-(3.18), in two stages:

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19. I am grateful to Geoff Lewis for pointing out to me that DH74's assumption (p9) that F is *strictly* concave is incompatible with linear homogeneity; but the assumptions here on f are all that are actually needed for our results.

LEMMA 3.1: *Along any Pareto-efficient path, the marginal product of capital (which equals, in a competitive economy, the interest rate) is strictly decreasing, and approaches* ρ, *i.e.*

$$\dot{F}_K(x(t)) < 0 \text{ for all } t;$$

$$F_K(x(t)) \to \rho \text{ as } t \to \infty.$$ [20]

PROOF: See Appendix 3.1.

LEMMA 3.2: *The PV-optimal consumption path* $\tilde{C}$ *must either be as shown in Figure 3.1 (i.e. first rising, and then falling asymptotically towards zero), or always falling towards zero.*

PROOF:

From DH74 (p11), the PV-optimal path $\tilde{C}(t)$ obeys the Ramsey savings rule:

$$\dot{\tilde{C}}/\tilde{C} = (\tilde{F}_K - \delta)/\eta(\tilde{C}) \quad \text{for all } t, \tag{3.19}$$ [21]

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20. Strictly we should write $f'(x)$ instead of $F_K(x)$, since F is formally a function of K and R; but we retain F_K here and in the Appendices as a reminder that it is the marginal product of capital.

21. A comment by David Ulph led me to study a variant of PV-optimality which highlights the contrast between the weighting approach mentioned in Chapter 2 and sustainedness as non-declining utility. Suppose that the economy's objective function is $\int_0^\infty e^{-\delta t} w(t) U(C(t))dt$ instead of $\int_0^\infty e^{-\delta t} U(C(t))dt$, where w is some intertemporal weighting function. Then it is straightforward to show that (3.19) must be replaced by $\dot{C}/C = (F_K - \delta + \dot{w}/w)/\eta(C)$. A function $w(t) \propto\!+ \exp\{\delta t - \int_0^t f'[x(z)]dz\}$ (the sign $\propto\!+$ means 'positively proportional to) would then make $\dot{C} = 0$ ($\Rightarrow$ non-declining utility) on the (modified) PV-optimal path. This $w(t)$ is the product of $e^{\delta t}$, which cancels the negative exponential discount factor $e^{-\delta t}$ that drives the conventional PV-optimal path to zero asymptotically, and $\exp\{-\int_0^t f'[x(z)]dz\}$, which is just as strong as a discounting of future utility as the economy's production function can withstand, without leading to declining utility on the modified PV-optimal path. But given (3.16), such a $w(t) \to \infty$ as $t \to \infty$, so $\int_0^\infty w(t)dt$ does not converge. Hence $w(t)$ is not a normal weighting function.

where $\eta(C) := -U_{CC}C/U_C$, the elasticity of marginal utility, is positive (but not necessarily constant) by (3.12). So if the initial productivity of capital $F_K(\tilde{x}(0)) > \delta$, $\tilde{C}$ is initially rising. But by Lemma 3.1, $(\tilde{F}_K - \delta)$ always declines over time and approaches $-(\delta - \rho)$ (<0, by (3.16)) in the long run. So the growth rate of $\tilde{C}$ given by (3.19) also approaches a negative number bounded away from zero, and $\tilde{C}$ itself must be asymptotically zero. If $F_K(\tilde{x}(0)) < \delta$, the initial rising phase does not occur. ||

As its last line shows, this proof has not established that the PV-optimal path is single-peaked rather than always falling; only that if consumption is initially rising, then its growth rate always falls and eventually becomes negative, giving at most a single peak. Let us turn then to the question of initial sustainability (which also implies single-peakedness). That is, when is $U_0^m \geq \tilde{U}(0)$? A problem here is that the general conditions for sustained paths to exist *at all* are difficult to establish, and like Asheim (1994) I have nothing to add to the analysis of Cass and Mitra (1979). For a specific example of both single-peakedness and initial sustainability, however, we can resort to a special case of the economy in (3.9)-(3.18):

The Special Case:

$$F(K,R) = K^{\alpha}R^{1-\alpha}; \quad U(C) = C^{1-\alpha}/(1-\alpha); \quad 0.5 < \alpha < 1 \text{ [22]} \tag{3.20}$$

Appendix 3.2 shows that the initial PV-optimal and maximin consumption levels for this case are:

$$\tilde{C}(0) = \delta K_0/\alpha; \quad C_0^m = \alpha\{K_0^{2\alpha-1}[(2\alpha-1)S_0]^{1-\alpha}\}^{1/\alpha}.$$

For low enough δ, clearly $U[\tilde{C}(0)] < U(C_0^m)$, i.e. the PV-optimal utility path is initially sustainable. Note also that $C_0^m/\tilde{C}(0) \propto K_0^{(\alpha-1)/\alpha}$ and

22. $0.5 < \alpha$ is the Solow (1974) condition for positive, permanently constant consumption to be feasible.

$\alpha - 1 < 0$, so increasing the initial capital stock K_0 makes initial sustainability *less* likely, contrary to the assertion in DH74 (p17). Intuitively, a large stock of capital will decrease its marginal product and hence the incentive to save. Exploring the Special Case by spreadsheet simulations was the inspiration for much of this chapter, and we will use the Special Case throughout Section 3.5.

3.3.2 Doubts about the neoclassical framework

The above analysis raises doubts about the validity of using a neoclassical framework like (3.9)-(3.18) to study sustainability, which are worth recording here before moving on to study opsustimality, the central theme of the second half of this chapter.

Firstly, the assumption of *constant returns* in just capital K and resource flow R, which comes from DH74 and is retained for the rest of this Chapter, is in some sense inappropriate. Labour clearly does play a major role in production, which ought to imply diminishing returns in capital and resources, as DH79 (p199) recognise. However, this seems at root to be a mathematical rather than a physical problem, since the properties that follow from the constant returns assumption, such as single-peakedness and initial sustainability, are still intuitively appealing in the diminishing returns case. Though to my knowledge these properties have not yet been proved for diminishing returns, they might be in future.

More troubling is the physical unreality of the infinite time horizon assumed, as already noted at the end of Section 1.3 of Chapter 1, which leads to excessively gloomy results about the effect of finite resources on sustainability. An approach perhaps worth trying, but not explored here, would be to use a discrete generation approach with a zero terminal value for the resource stock at the end of the last generation. However, the omens for tractability are not good, since Howarth's work with a finite number of generations which was noted in Section 1.3 always ends up with numerical solutions, even for just two overlapping generations.

Also troubling, but in the direction of excessive optimism, is the ability of unending capital-resource substitution in the Cobb-Douglas production function to produce an unbounded amount of valued output from a finite physical input of natural resources – output which would be what Daly (1977, 118) calls "angelized GNP". This is also physically unrealistic, because it surely breaks the First Law of Thermodynamics: the average product of resources, F/R, is surely bounded above (which also rules out the assumption of endless technical progress made in Chapter 2). This observation is at the heart of the ecological economic critique by Daly, Pearce, and several others, of the neoclassical approach to sustainability, which has already been noted in Section 1.2.2. But note the above 'surely' qualifiers: proving that F/R is bounded involves human psychology as well as thermodynamics, a problem often ignored by the ecological school of thought. A rigorous proof would require extensive work on what people derive value from, as well as the ultimate limits to technical progress in producing value.

So, contrary to Solow (1974, p34), the most realistic case to investigate theoretically is the combination of a finite time horizon with a bounded average product of resources (which would mean an elasticity of substitution less than 1, if it is constant); but this remains for further work. Our main concern here is to demonstrate the technical properties of opsustimality in the simplest way, so we stick to the well-known neoclassical assumptions, despite their physical inexactitudes.

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Section 3.4 now considers analytically what the opsustimal path looks like in a fairly general case, and what happens to aggregate wealth and aggregate investment on it. Section 3.5 uses the above Special Case to show analytically when the opsustimal path is distinct from the maximin, and numerically how it compares to the PV-optimal path and how it is affected by changes in parameter values and in the initial consumption level. Section 3.6 returns to the general case of Section 3.3.1 to analyse what policy interventions could convert a PV-optimal path into an opsustimal one.

3.4 OPSUSTIMALITY I: GENERAL RESULTS

In this Section we derive two key results for the opsustimal consumption path, denoted $C^{\dagger}(t)$.[23] This is the solution of the PV-optimality problem (3.9)-(3.18) in the Dasgupta-Heal economy, subject to the constraint that utility (and therefore consumption) must be sustained:

$$\text{For all } t_1,\ t_2 \geq 0,\ \ t_2 > t_1 \Rightarrow C(t_2) \geq C(t_1); \tag{3.21}$$

or if consumption is differentiable, $\dot{C} \geq 0$ for all t. Assuming that this constraint is feasible for strictly positive consumption, Section 3.4.1 shows that the opsustimal path has either rising consumption followed by constant consumption, or constant consumption forever. Section 3.4.2 then shows that on the opsustimal path, non-declining aggregate wealth does correspond to non-declining utility, unlike on the PV-optimal path.

23. Rather than using Chapter 2's tilde/cross combination (as in $\tilde{C}^{\dagger}$) for opsustimality, here we use just the cross superscript †, for visual simplicity.

3.4.1 The two-phase nature of the opsustimal path

PROPOSITION 3.7: *The opsustimal consumption path is continuous, and either has constant consumption (and hence utility) for all t, or corresponds to path $U^{\dagger}(t)$ in* ***Figure 3.4****, which has rising utility for* $0 \leq t < T^{\dagger}$*, and constant utility for* $t > T^{\dagger}$.

In essence, this result has already been proved by Asheim (1988, Lemma 4). But he used a production function with diminishing returns to scale and discrete time, and discrete time makes some important conclusions about continuity less transparent. So we present a separate proof here for a constant-returns economy in continuous time, which also helps to build intuition about the nature of opsustimality, by way of Lemma 3.2 from the last section and three further Lemmas. The full proofs of these are in Appendix 3.3, although we give a few hints here.

LEMMA 3.3: *The opsustimal consumption path cannot always rise.*

(The intuition here is that rising consumption in the long run, even if feasible, would 'overachieve' sustainedness. The eventual decline of the PV-optimal path means that the PV of an always rising path could be increased, without breaking the sustainedness constraint, by shifting consumption earlier in time until the path is no longer rising in the long run.)

LEMMA 3.4: *On a continuous opsustimal consumption path, a period with* $\dot{C}=0$ *cannot be followed by a period with* $\dot{C}>0$.

(If it is, saving more at the start of the period with $\dot{C}=0$ and spending it when $\dot{C}>0$ can increase PV, without breaking the sustainedness constraint.)

LEMMA 3.5: *The opsustimal consumption path is continuous.*

(This follows from the strict concavity of the utility function.)

Using the above Lemmas, we can give:

PROOF OF PROPOSITION 3.7:

At $t = 0$, a opsustimal path must have either $\dot{C}=0$ or $\dot{C}>0$ (since $\dot{C}<0$ breaks the sustainedness constraint). If it has the former, then by Lemma 3.4, $\dot{C}$ must remain zero for all time. If it has the latter, then by Lemma 3.3, $\dot{C}$ must become zero after some finite time, and then by Lemma 3.4, must remain zero for the rest of time. Lemma 3.5 rules out upward discontinuities between separate segments with $\dot{C}=0$. ||

3.4.2 Aggregate wealth and sustainability on the opsustimal path

We first state two Lemmas on continuity on the opsustimal path, which are both proved in Appendix 3.4. Note, for use in the proof of Lemma 3.7 and later, that in a competitive Dasgupta-Heal economy, prices $\pi=(U_C,U_CF_R)$ and stocks $\Sigma=(K,S)$, so aggregate investment $\pi.\dot{\Sigma}=U_C(\dot{K}-F_RR)$.

LEMMA 3.6: *The opsustimal resource flow rate R is continuous.*

LEMMA 3.7: *Aggregate investment is continuous on the opsustimal path, and zero from $T^{\dagger}$ onwards. Net national welfare is continuous on the opsustimal path, and equal to utility from $T^{\dagger}$ onwards.*

Using these Lemmas, we can prove the following result, illustrated by Figure 3.4 and based on the two phases identified by Proposition 3.7.

PROPOSITION 3.8: *Let the transition time between the rising and constant utility phases on the opsustimal path be $T^{\dagger}$. Then:*

$$t<T^{\dagger} \;\Rightarrow\; U^{\dagger}(t) \;<\; U^{m\dagger}(t) \;<\; Z^{\dagger}(t) \;=\; \delta\Psi^{\dagger}(t) \;=\; \delta PV_t(U^{\dagger})$$

$$t\geq T^{\dagger} \;\Rightarrow\; U^{\dagger}(t) \;=\; U^{m\dagger}(t) \;=\; Z^{\dagger}(t) \;=\; \delta\Psi^{\dagger}(t) \;=\; \delta PV_t(U^{\dagger})$$

and all five functions are continuous at $t=T^{\dagger}$.

PROOF: See Appendix 3.4.

This proposition differs significantly from Proposition 3.6 in Section 3.2.1, which was for the combination of the PV-optimal path and an unanticipated constant utility phase. This is because the policy which achieves the constant phase of the opsustimal path from $T^{\dagger}$ onwards must have been *pre-announced* at time zero, so that private agents could optimise PV in response to the policy and make a continuous transition to the constant phase at time $T^{\dagger}$.

A notable aspect of Proposition 3.8 is that for $t<T^{\dagger}$, $\mathrm{PV}_t(U^{\dagger})$, the forward-looking measure of wealth, is the same as both the backward-looking measure of wealth $\Psi^{\dagger}$ and the instantaneous measure $Z^{\dagger}/\delta$, in spite of the transition to the constant consumption phase after $t=T^{\dagger}$.[24] However, current maximin utility $U^{m\dagger}$ is still below net national welfare $Z^{\dagger}$ during the rising phase. Also notice that, since aggregate investment is net national welfare $Z^{\dagger}$ minus current utility $U^{\dagger}$, it is always positive (i.e. aggregate wealth is always rising) before $T^{\dagger}$. So on the opsustimal path aggregate wealth and utility both rise together, and then both are constant together. But although rising aggregate wealth thus guarantees sustainability, this is of little practical value for detecting *un*sustainability, unless the prices π that

24. Note also the analogy between $Z^{\dagger}=\delta\Psi^{\dagger}$ here and Solow's claim, discussed in Section 3.2.2, that $Y=rV$.

would hold on the opsustimal path can somehow be estimated from current, 'unsustainable' prices and other data.

3.5 OPSUSTIMALITY II: SPECIAL AND NUMERICAL RESULTS

Here we tackle a number of questions which can be answered analytically or numerically in the Special Case of Section 3.3.1, but not at all in the more general case there. The results are intuitively appealing, and I conjecture that they may be true for a wider range of economies than just the Special Case.

3.5.1 When does the rising phase of the opsustimal path occur?

Does the rising phase of the opsustimal path, identified as a possibility by Proposition 3.7, actually exist? The answer is simple for the Special Case:

PROPOSITION 3.9: *If $\delta(K_0/S_0)^{(1-\alpha)/\alpha} < (2\alpha-1)^{1/\alpha}$, the opsustimal path is different from the maximin path in the Special Case. If $\delta(K_0/S_0)^{(1-\alpha)/\alpha} \geq (2\alpha-1)^{1/\alpha}$, the opsustimal and maximin paths are the same.*

PROOF: See Appendix 3.5. ||

By definition, if the opsustimal path is different from the maximin path, it must have higher PV than the maximin. So Proposition 3.9 shows that in the Special Case, there is no scope for getting higher PV than the maximin path while maintaining sustainedness if impatience (δ) or the initial capital/resource stock ratio (K_0/S_0) is too high. Also, comparing (A3.15) and (A3.16), from Appendix 3.2, $\delta(K_0/S_0)^{(1-\alpha)/\alpha} \lesseqgtr (2\alpha-1)^{1/\alpha} \Leftrightarrow C_0^m \gtreqless [\tilde{C}(0)]\alpha^2/(2\alpha-1)$ $(>\tilde{C}(0)$, since $\alpha>0.5)$. Differentiating (A3.13) shows that $\dot{\tilde{C}}(0)>0 \Leftrightarrow \delta<\alpha(S_0/K_0)^{(1-\alpha)/\alpha}$. **Table 3.1** summarises the four possible initial states of the Special Case economy that then arise. As

Table 3.1 Initial opportunities for a transition to sustainedness in the Special Case economy

Range of $\Theta := \delta(K_0/S_0)^{(1-\alpha)/\alpha}$	Opsustimal path different from (and hence higher PV than) the maximin path?	PV-optimal path initially sustainable?	Initial PV-optimal consumption rising?
$\Theta < (2\alpha-1)^{1/\alpha}$	Yes	Yes	Yes
$(2\alpha-1)^{1/\alpha} \leq \Theta \leq \alpha^2(2\alpha-1)^{(1-\alpha)/\alpha}$	No	Yes	Yes
$\alpha^2(2\alpha-1)^{(1-\alpha)/\alpha} < \Theta < \alpha$	No	No	Yes
$\alpha \leq \Theta$	No	No	No

$\delta(K_0/S_0)^{(1-\alpha)/\alpha}$, denoted by Θ, rises[25] (whether through a higher utility discount rate δ, a higher initial capital stock K_0 or a lower initial resource stock S_0), the initial opportunities for making a painless transition from a PV-optimal to a sustained economy are successively extinguished. First the opsustimal path can no longer improve on the maximin path; then the maximin is lower than the initial PV-optimal consumption, so the latter is unsustainable; finally, the PV-optimal path itself has falling consumption from the start.

This succession also happens simply as time passes on a PV-optimal path during its rising phase, if it has one, since from (A3.13) and (A3.14), $\widetilde{K}/\widetilde{S} \propto \{(1-\alpha)t+[\widetilde{x}(0)]^{1-\alpha}\}^{1/(1-\alpha)}$ which strictly increases over time. So even if the prospects for moving from PV-optimality to sustainedness are

25. Note that $\frac{1}{2} < \alpha < 1 \Rightarrow 0 < (2\alpha-1)^{1/\alpha} < \alpha^2(2\alpha-1)^{(1-\alpha)/\alpha} < \alpha$.

initially cheerful, they become grimmer as time goes on. To the extent that the model here is anything like a parable of the real world, and assuming that a sustainedness constraint is contemplated but not yet in force, it would obviously be of great interest to policy-makers to know roughly in which of the four states of Table 3.1 the world currently lies.

3.5.2 Is opsustimal consumption initially higher than PV-optimal consumption?

A result which turns up in numerical solutions of the Special Case is that the opsustimal consumption path is initially higher than the PV-optimal consumption path. Since the first path is sustained while the second is not, this may seem quite counterintuitive. However, it can be explained by considering what happens to aggregate investment on the two paths. Proposition 3.5 showed that at the last time when a PV-optimal path is sustainable, aggregate investment is still positive (as shown at point G at time T_L on Figure 3.2), and would then drop to zero on the constant utility path that starts from G (by Hartwick's Rule). But from Lemma 3.7 and Proposition 3.8, aggregate investment is continuous on the opsustimal path and zero at the change to constant utility (which will, however, happen at a different time $T^{\dagger}$). This suggests that aggregate investment is *lower* on the opsustimal path than on the PV-optimal path at the same time, and the intuition for this is that the PV-optimal path 'overinvests' initially when returns are high, only to disinvest ($\dot{K}<0$) later when returns become too low. But *ceteris paribus* aggregate investment $U_C[F_K(K,R)K-C]$ is decreased by higher consumption C, or (assuming $F_{KR}>0$) by lower resource flow R. So initially we expect to find higher C and lower R on the opsustimal path.

As noted above, concrete evidence on the initial behaviour of C and R is available only from numerical solutions of the Special Case of Section 3.3.1. The reason why the opsustimal path of this case cannot be found analytically is that there is no exogenous terminal condition for the rising phase, assuming that one occurs. Equation (A3.11) therefore cannot be integrated definitely, and we then have no analytic expression for $S^{\dagger}(T^{\dagger})$, the resource stock at the end of the rising phase. However, from the equations (A3.7) and (3.10) for the production and distribution of output, we do have a differential equation for K in the opsustimal rising phase:

$$\begin{aligned}\dot{K} &= K^{\alpha}R_0{}^{1-\alpha}\{1-[\alpha C(0)/K_0\delta](1-e^{-(\delta/\alpha)t})\}^{1-\alpha} \\ &- [C(0)/x(0)]\{(1-\alpha)t+[x(0)]^{1-\alpha}\}^{1/(1-\alpha)}e^{-(\delta/\alpha)t}\end{aligned}$$

Using numerical methods, we can integrate this, and (A3.11) for $\dot{R}$. Then we apply the continuity condition at $T^{\dagger}$ with the maximin path starting then, which is found by substituting $K^{\dagger}(T^{\dagger})$ and $S^{\dagger}(T^{\dagger})$ for K_0 and S_0 in (A3.16). Thus we can compute the two-phase opsustimal path of the Special Case in some illustrative cases. We have in fact already seen the path for $\alpha=0.7$, $\delta=0.1$, $K_0=S_0=1$ in Figure 3.4 (the same parameter values as were used for Figures 3.2 and 3.3). **Figure 3.5** now illustrates all the above by plotting to scale the PV-optimal, initial maximin and opsustimal paths of (a) consumption and aggregate investment[26], (b) resource flow and (c) capital stock in this case. Limited numerical experimentation has so far shown this Figure to be a typical case, provided that the opsustimal path has an initial rising phase (so $\Theta<(2\alpha-1)^{1/\alpha}$, from Table 3.1). I therefore suggest:

26. Figure 3.5 uses consumption rather than utility units simply because the visual comparison of the PV-optimal and opsustimal paths is more striking. 'Aggregate investment' as plotted here is therefore $\dot{K}-RF_R$ rather than $U_C(\dot{K}-RF_R)$.

CONJECTURE 3.1:

Provided that $\Theta = \delta(K_0/S_0)^{(1-\alpha)/\alpha} < (2\alpha - 1)^{1/\alpha}$ *in the Special Case, or that an equivalent condition holds in a more general case:*

(i) *Opsustimal consumption* $C^\dagger$ *starts higher than PV-optimal consumption* $\widetilde{C}$ *and lower than initial maximin consumption* C_0^m*;*

(ii) *Opsustimal resource flow* $R^\dagger$ *starts out lower than both PV-optimal and maximin flows.*

If there is a rising opsustimal phase and (i) holds, then the constant phase of $C^\dagger$ must be higher than the initial maximin C_0^m, otherwise $C^\dagger$ could not have higher PV than C_0^m. And if (ii) also holds, opsustimal aggregate investment, which equals $F_K(K,R)K - C$ by the proof of Lemma 3.7, is initially lower than PV-optimal investment, but higher of course than maximin aggregate investment (which is zero, by Hartwick's Rule).

3.5.3 How do changes in parameters and in the initial consumption level affect the opsustimal path?

To explore how changes in key *parameters* affect the opsustimal path, **Table 3.2** lists some approximate results for changes in the utility discount rate, and the capital and resource stocks, in the Special Case for the parameter values noted, and using the notation developed above. (The results are approximate because powerful numerical methods would be needed to compute the opsustimal path very accurately, and the manual spreadsheet iterations used can give results only to two or three significant figures.)

It seems intuitively likely that the direction of the changes shown in the Table hold for other parameter values as long as $\Theta < (2\alpha - 1)^{1/\alpha}$, and indeed in a more general economy altogether. So for example, a higher utility discount rate should raise initial consumption and resource depletion on the

Table 3.2 Selected numerical results for opsustimal paths

	Case	1: Base	2: Higher discount rate δ	3: More initial resource S_0	4: Less K_0, more S_0
Para-meters	K_0	1	1	1	0.738[1]
	S_0	1	1	1.5	1.5
	α	0.7	0.7	0.7	0.7
	δ	0.1	0.12	0.1	0.1
Re-sults	$C^{\dagger}(0)$	0.15	0.18	0.15	0.11
	$C^{\dagger}(0)/\widetilde{C}(0)$	1.07	1.08	1.07	1.06
	$C^{\dagger}(T^{\dagger})$	0.74	0.65	1.01	0.94
	$C^{\dagger}(T^{\dagger})/C_0^m$	1.57	1.38	1.79	2.00
	$R^{\dagger}(0)$	0.13	0.16	0.20	0.21
	$R^{\dagger}(0)/\widetilde{R}(0)$	0.94	0.93	0.95	0.96
	$\dot{K}^{\dagger}(0)$	0.40	0.39	0.47	0.39
	$T^{\dagger}$	7.2	5.3	7.9	8.4

1. This value is chosen so that the initial maximin consumption level C_0^m is the same as for Cases 1 and 2.

opsustimal path; make the change to a constant consumption phase happen sooner; and lower consumption in the constant phase. But all this remains as another conjecture.

A different question is to ask what will be the effect on the opsustimal path of a *historically given initial consumption level C_G*. In Section 2.5 of Chapter 2 we explored this question in a model with utility $U(C,S)$ dependent on the resource stock as well as on consumption; we therefore had to look at the effects on sustainability of both C_G itself (questions (i) and (ii) in Section 2.5), and of a change in the initial resource stock S_0, given C_G (questions (iii) and (iv) there). Here utility $U(C)$ is independent of the resource stock, so we need look at only the first effect. **Figure 3.8** is a mixture of conjecture and certainty about what happens to the *constrained-*opsustimal (COS) consumption path in a general Dasgupta-Heal economy as given initial consumption C_G rises above the initial opsustimal level, $C^{\dagger}(0)$.

It is straightforward to show that Lemmas 3.3-5 and hence Proposition 3.7 still hold for the COS path, and hence that COS paths must generally comprise a rising phase followed continuously by a constant phase, as shown. Since these paths must be efficient, they must intersect with all other efficient paths such as each other, the PV-optimal path $\widetilde{C}(0)$H, the opsustimal path $C^{\dagger}(0)$A, and the maximin path C_0^mE as shown. I *conjecture* that the effect of a historical constraint which increases initial consumption above $C^{\dagger}(0)$, but only to a sustainable level (so that we are answering question (i)), will be similar to the effect which a sustainedness constraint was shown to have by using a numerical solution in the previous subsection. If so, C_1B, the COS path starting from $C_1 > C^{\dagger}(0)$, stays above $C^{\dagger}(t)$ until it (the COS path) becomes constant, as shown; and the COS path C_2D stays above C_1B until the former becomes constant, as shown; and so on until we reach the initial maximin consumption level C_0^m. The effect of a higher (but still sustainable) initial consumption level is then to shorten the duration of the 'growth window' during which consumption can rise before having to level off, which seems plausible.

However, if $C_G > C_0^m$, there is no sustainable path, and we are answering question (ii) in Section 2.5 of Chapter 2. What the 'best' development path is then depends on what we assume about the adjustment costs of returning consumption to a sustainable level. With adjustment costs dependent on the rate of consumption decline, I conjecture that the path will look something like C_3F, with a continuous decline in consumption followed by constant consumption at a level below C_0^m (as required by the efficiency of C_0^mE).

3.6 OPSUSTIMALITY III: POLICIES TO ACHIEVE IT

3.6.1 General analysis

Thanks to the assumption of constant returns to scale, we can regard the ownership of both the capital and resource stocks in our model as being divided equally among a large number of identical, price-taking agents. There are apparently no externalities in the model, so that Conjecture 3.1 from Chapter 2, that sustainedness can be a collective good if a non-renewable resource has social amenity value, does not apply. Why then is there is any role for public policy in achieving sustainedness here? If agents seek it, why cannot they simply change their private resource depletion and consumption plans accordingly? The answer is given at some length in Chapter 4. The underlying reality, which an infinitely-lived, representative agent model merely seeks to reflect, is that society comprises different generations and different sexes. Any parents' bequests of capital and resources to their children give an altruistic 'warm glow' not just to themselves, but also to the parents of the mates that their children select, so any degree of randomness in mate selection gives rise to intragenerational externalities. If individual parents care about sustainedness at the family level, sustainedness may then be a valid goal of public policy, because the mixing of bequests that happens when children mate make it infeasible, or at least inefficient, for parents achieve sustainedness by their own actions. So we assume here that agents seek unconstrained PV-maximisation in their private choices, yet still vote for a government which enacts non-coercive (i.e. market-based) policies to achieve sustainedness.

Chapter 2 explored sustainedness policies (which have been neglected by the literature on Hartwick's Rule) in detail, so we will be somewhat briefer here. We assume that policymakers influence private saving and resource depletion decisions by specific taxes (or 'fees') of $\phi_C(t)$ on consumption, $\phi_K(t)$ on capital, $\phi_R(t)$ on resource depletion, and $\phi_S(t)$ on the resource stock (which will in practice be a negative tax, i.e. a subsidy to encourage conservation of the remaining stock). All tax revenues are returned to agents as lump sums which are not affected by any one agent's choices. The undiscounted Hamiltonian of the unconstrained control problem in (3.9)−(3.17) then becomes, from the point of view of an individual,

$$H = U(C) + \pi_K(t)[F(K,R) - C(1+\phi_C) - \phi_K K - \phi_R R - \phi_S S] - \pi_S(t)R$$

From Chapter 2, Section 2.4.1, we can see[27] that the consumption $\hat{C}$ and capital/resource ratio $\hat{K}/\hat{R} = \hat{x}$ that will be chosen by individuals in response to the taxes respectively obey the following modified Ramsey rule and modified Hotelling rule:

$$\dot{\hat{C}}/\hat{C} = [\hat{f}' - \delta - \dot{\phi}_C/(1+\phi_C) - \phi_K]/\eta \tag{3.22}$$

$$\dot{\hat{x}}/\hat{x} = (1 - \phi_K/\hat{f}')\hat{\sigma}\hat{f}/\hat{x} - (\hat{f}'\phi_R - \dot{\phi}_R - \phi_S - \phi_K\phi_R)/\hat{x}^2(-\hat{f}'') \tag{3.23}$$

Between them, (3.22) and (3.23) define the range of tax schedules $\phi_C(t)$, $\phi_K(t)$, $\phi_R(t)$ and $\phi_S(t)$ by which the government might cause private agents to choose constant consumption paths ($\dot{\hat{C}} = 0$). We consider first the role of the consumption tax and the capital tax, and then the two resource taxes.

3.6.2 A consumption tax or capital tax must become a subsidy, and resource taxes are powerless

Consider first the case where the opsustimal path has an initial phase of rising consumption ending at time $T^\dagger$. Any tax which alters intertemporal allocation during this phase will cause consumption to depart from local PV-

27. By ignoring the technical productivity factor and all derivatives of utility with respect to the resource stock there, which are features absent here.

optimality, which could not be opsustimal. So from (3.22), any *consumption* tax has to be at a constant level ($\dot{\phi}_C=0$), or a *capital* tax must be zero ($\phi_K=0$)[28] during a rising phase. Continuity of consumption, and the possibility of arbitrage across the transition time $T^\dagger$, mean that either tax must be at the same level just after $T^\dagger$.

But after $T^\dagger$, the consumption tax cannot stay constant (or, the capital tax cannot stay zero). Approaching time $T^\dagger$ at the end of a rising phase, (3.19) implies $f'(x)>\delta$. Resources are not taxed, so arbitrage will keep the resource price $F_R(K,R)$ and hence resource flow R and $x=K/R$ continuous across time $T^\dagger$, as in Lemma 3.6; so $f'(x)>\delta$ just after $T^\dagger$. To get $\dot{C}=0$ thereafter, from (3.23) $\dot{\phi}_C$ or ϕ_K initially need to be positive, and then (since $f'\to\rho<\delta$ as $t\to\infty$, by Lemma 3.1) negative; effectively, they lower the utility discount rate that people use to calculate their optimal saving. During the constant consumption phase, a consumption tax ϕ_C must therefore first rise, and then fall asymptotically to a 100% subsidy on consumption (since we must have $\lim|_{t\to\infty}\dot{\phi}_C/(1+\phi_C) = -(\delta-\rho)$ and thus $\lim|_{t\to\infty}\phi_C = -1$, by Lemma 2.1 in Chapter 2); whereas a capital tax must fall to $-(\delta-\rho)$, a subsidy equal to the difference between the interest rate ρ and the utility discount rate.

Intuitively, the tax can be regarded as 'squashing flat' the unsustainable peak of the PV-optimal utility curve in Figure 3.1 and elsewhere, bringing some of it forward with a rising ϕ_C or positive ϕ_K, and then pushing back the rest with a falling ϕ_C or negative ϕ_K. Although this is no problem in theory, in practice it would be near-impossible politically because of the lump-sum taxes needed to finance the subsidy phase, as noted in Chapter 2.

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28. There is little point in considering the case where $\dot{\phi}_C/(1+\phi_C)+\phi_K=0$ but $\dot{\phi}_C/(1+\phi_C)\neq 0\neq\phi_K$.

Also, the government's intention to achieve constant consumption from $T^\dagger$ onwards must be *credibly announced at time zero*. Only then will private agents be able to use their perfect foresight to depart from PV-optimality and choose the higher initial consumption and lower initial resource flow that (according to Conjecture 3.1) characterise the opsustimal path. If there is no initial phase of rising consumption, we have $f' < \delta$ at time zero, so the policy starts straightaway with no need for pre-announcement. The consumption tax falls (or the capital tax is negative) from time zero onwards, ending up with politically impractical subsidies as before.

Because they do not appear directly in (3.22), but influence the time path of consumption only through their effect on the capital/resource ratio x in (3.23), neither a *resource depletion* tax ϕ_R nor a *resource stock* tax ϕ_S (in practice a subsidy) can achieve (permanent) sustainedness. For consider (3.22) with $\phi_C = \phi_K = 0$. The only way that $\dot{C}$ can be zero from the transition time $T^\dagger$ onwards is for x to be held constant for all $t \geq T^\dagger$ at the level x_δ defined by $f'(x_\delta) = \delta$.[29] R is then proportional to K, and both must be bounded away from zero to support a constant consumption level; but such a non-vanishing R can be sustained for only a finite time by a finite stock S_0. So both a depletion tax and a stock subsidy are powerless to achieve sustainedness, given the assumptions of this chapter.

29. This is not true if there is exogenous technical progress: see Section 2.4.2 of Chapter 2.

3.7 CONCLUSIONS

In an economy with accumulating capital, non-renewable resources and constant technology, maximising the present discounted value of utility ('PV-optimality') can be unfair to the future, by subjecting distant generations to misery, even though rising wellbeing is possible. But constant utility (the result of the maximin criterion) is often an inefficient way of being fair to the future, because it bans current investment which would be preferred by both current and future generations. A third criterion, that of maximising PV provided that utility never declines – called PV-optimal sustainedness or 'opsustimality' here – offers an intuitively attractive compromise which can perhaps be both fair and efficient.

My primary purpose in this chapter has not been to inquire into the philosophical foundations of opsustimality, but to analyse its purely technical properties. What does an opsustimal development path look like? When is it distinct from the maximin path, and how does it compare to the PV-optimal path? What policies might achieve it?

A related, prior question is whether there is some measurable condition such as 'maintaining aggregate wealth' or 'keeping total capital constant' which guarantees sustainability. The theory of such measures seemed a natural place to look for tools for analysing opsustimality, and so it proved. But, in common with Asheim (1994), we found an important flaw. If it is measured at prices which are corrected for obvious distortions like monopoly and pollution, rising aggregate wealth (defined as the integral over time of aggregate investment in capital and resource stocks) does not guarantee sustainability. Indeed, in a typical PV-optimal resource economy with a sustainability problem, there is bound to be a period when rising wealth and unsustainability happen at the same time. This is because an unsustainable economy depletes non-renewable resources too rapidly, and drives down their prices. Aggregate wealth measured at such prices can then still be rising, which gives a false reassurance about sustainability.

A formal investigation of this problem and its offshoots took up about half the chapter. It yielded a number of other results about welfare and wealth measured on an initially sustainable, PV-optimal utility path, such as:

(a) net national welfare (current utility plus the rate of increase of aggregate wealth) cannot be attained as a permanent utility level;
(b) declining aggregate wealth does imply *un*sustainability;
(c) waiting until aggregate wealth is at a peak before making a transition to the maximum available constant utility path does not sustain development, because it results in a downward jump in utility and net national welfare at the transition time, although not in aggregate wealth itself.

We then clarified a number of misinterpretations of these results that exist in the literature, particularly in the influential paper by Solow (1986) which overlooks points (a) and (c). And in order to show that an initially sustainable PV-optimal path exists, and thus justify the assumptions that underlie result (c), we first had to analyse a more restrictive 'Dasgupta-Heal economy' with one capital and one non-renewable resource stock, and constant returns to these two stocks, and then a Special Case of this economy with Cobb-Douglas production and isoelastic utility. This raised more general doubts about the relevance to sustainability modelling of the infinite time horizon and infinite capital-resource substitutability typically assumed in neoclassical growth modelling.

Moving on to opsustimality itself, several results were established, although at differing levels of generality, as summarised in **Table 3.3**,

Table 3.3 Summary of main results, and their level of generality

Results obtain-ed	Summary of their meaning	Assumptions made (and hence level of generality)
Propo-sitions 3.1-3.4	Net national welfare is equivalent constant utility level of PV, and therefore not achievable as constant utility; net national welfare is return on aggregate wealth; declining aggregate wealth implies unsustainability.	1: Convex, smooth, regular, capital-resource economy.
Propo-sitions 3.5-3.6	There must be a period when wealth is rising on PV-optimal path, but path is unsustained.	2: As 1, but also initially sustainable and single-peaked.
Propo-sitions 3.7-3.8. Section 3.6	Opsustimal consumption (and utility) path has (perhaps) rising phase followed by constant phase. Opsustimal net national welfare is return on aggregate wealth, equals equivalent constant utility of PV. Consumption or capital tax must become subsidy to sustain constant utility; resource taxes cannot sustain constant utility.	3: As 2, but also one capital, and one resource stock, constant returns, low asymptotic marginal productivity of capital.
Appen-dix 3.1, Propo-sition 3.9	Low enough discount rate or initial capital/resource stock ratio makes PV-optimal utility initially sustainable, and (at a lower threshold) makes opsustimal path different from maximin path.	4: As 3, but also exact functional forms (i.e. the Special Case); analytic result.
Conjec-ture 3.1, Table 3.2	Consumption is higher on initial rising phase of opsustimal path than on PV-optimal path. Higher discount rate raises initial opsustimal consumption, lowers transition time and eventual opsustimal consumption, etc.	5: As 4, but numerical result

which includes the above results on aggregate wealth. For the general Dasgupta-Heal economy, we showed that the opsustimal path either comprises a phase of rising consumption (and therefore utility) followed by a phase of constant consumption, or has constant consumption always. All the economy's variables are continuous between the two phases, in contrast to the discontinuities in (c), and the correspondence between non-declining aggregate wealth and non-declining utility is exact. On the opsustimal path, net national welfare is always both the return on aggregate wealth, and the constant utility equivalent of PV.

The exact solution of the Special Case showed that the opsustimal path differs from (and therefore has higher PV than) the initial maximin path only if impatience or the initial capital/resource stock ratio is low enough. Since this ratio rises without limit as the PV-optimal path develops, the current opsustimal path will eventually merge into the current maximin. Numerically, we found that the rising phase of an opsustimal path has slightly a lower resource flow but *higher* consumption than the PV-optimal path, which is an attractive feature, and accords with the intuition that the PV-optimal level of capital investment is initially too high for sustainability.

Finally, we returned to the more general Dasgupta-Heal economy, and revisited the conclusion in Chapter 2 that it is surprisingly hard to find an effective sustainedness policy, given that consumption and resource depletion cannot be directly controlled in a market economy. To achieve the opsustimal path, a consumption or capital tax must be credibly announced at time zero, in order to shift the economy from the PV-optimal to the rising phase of opsustimal path (assuming there is one), and come into operation at the start of the constant phase. Both taxes must end up as subsidies, with all the political problems that financing them would entail in practice. Resource taxes are even worse, being unable to achieve sustainedness, essentially because they can shift resource depletion to the future, but cannot induce sufficiently high saving. The root of this long-term policy problem is clearly that attaining constant utility is plain difficult, once PV-maximising individuals have piled up capital and eaten away the resource to the point where they have a strong incentive to consume capital, rather than accumulate more of it.

I hope I have shown why indicators of sustainability are not always what they seem, and why opsustimality is an interesting and attractive, although not always straightforward, alternative to PV-optimality or to the maximin as a criterion of intergenerational equity. A natural extension of both topics would be to include other standard features of environmental and resource economics, such as renewable or repairable resources, resource stock effects on production, population growth, and extraction costs. But in further research, I would perhaps be tempted first to explore four issues, either separately or together, that were touched on above: irreversible capital accumulation; a finite time horizon; a production function with a bounded average product of resources; and last but not least the role of technical change, which can so radically transform any result concerning sustained or constant utility paths. Another thrust of theoretical research in this area should be to see whether anything useful can be said about the 'sustainability prices' which would make a change in aggregate wealth an accurate measure of sustainability. Or can we say only that if our current development is unsustainable, then the existing measure will warn us about it – but too late to prevent at least some suffering?

APPENDICES TO CHAPTER 3

Appendix 3.1 *Proof of Lemma 3.1*

LEMMA 3.1: *Along any Pareto-efficient path, the marginal product of capital (which equals, in a competitive economy, the interest rate) is strictly decreasing, and approaches* ρ, *i.e.*

$$\dot{F}_K(x(t)) < 0 \text{ for all } t; \tag{A3.1}$$

$$F_K(x(t)) \to \rho \text{ as } t \to \infty. \tag{A3.2}$$

PROOF:

Any efficient path in a capital-resource economy with no resource renewability and no externalities obeys Hotelling's Rule, $\dot{F}_R/F_R = F_K$. Given (3.17), resource flow is always strictly positive, by DH74's Proposition 5:

$$R(t) > 0 \text{ for all } t \text{ on any efficient path} \tag{A3.3}$$

and Hotelling's Rule then takes the special form (see DH74 p11):

$$\dot{x} = \sigma(x)f(x) \text{ for all } t \text{ on any efficient path} \tag{A3.4}$$

By (A3.3), x must be finite at $t=0$, and by (A3.4), x cannot be zero at $t=0$ (otherwise, by (3.14), R would remain infinite for a finite time, which is impossible with a finite resource stock). Hence by (A3.4) and (3.18),

$$\dot{x} \geq \bar{\sigma} f[x(0)] > 0 \text{ for all } t, \tag{A3.5}$$

$$\Rightarrow \quad x \to \infty \text{ as } t \to \infty.^{1} \tag{A3.6}$$

(A3.5) and (3.15) give (A3.1). (A3.6) and (3.16) give (A3.2). ||

1. Note the role here, and often elsewhere, of the infinite time horizon, the physical unreality of which was highlighted in Section 3.3.2.

Appendix 3.2 *Proof that PV-optimal utility in the Special Case can be initially sustainable and single-peaked*

DH74 (p17) studied the economy defined by (3.9)-(3.18) and in addition:

$$\left.\begin{aligned} U(C) &= C^{1-\eta}/(1-\eta), \quad \eta > 0 \\ F(K,R) &= K^{\alpha}R^{1-\alpha}, \quad 0.5 < \alpha < 1 \end{aligned}\right) \qquad \text{(A3.7)}$$

By defining $x:=K/R$, the capital-resource flow ratio, DH74 showed by integrating the Hotelling rule (which is $\dot{x}=x^{\alpha}$ here) that

$$\tilde{x}(t) = \{(1-\alpha)t+[\tilde{x}(0)]^{1-\alpha}\}^{1/(1-\alpha)} \qquad \text{(A3.8)}$$

and thus from the Ramsey rule $\dot{\tilde{C}}/\tilde{C}=(\tilde{F}_K-\delta)/\eta$ that PV-optimal consumption is

$$\tilde{C}(t) = \tilde{C}(0)[\tilde{x}(0)]^{-\alpha/\eta}\{(1-\alpha)t+[\tilde{x}(0)]^{1-\alpha}\}^{\alpha/\eta(1-\alpha)}e^{-(\delta/\eta)t}, \qquad \text{(A3.9)}$$

We cannot prove that this is initially rising (and hence that $\tilde{U}$ is also) unless we know $\tilde{x}(0)$ as an explicit function of the parameters K_0, S_0, α and δ, but this seems impossible to calculate. However, since $\dot{R}=-C/x$ in an efficient Cobb-Douglas economy,[2]

$$\dot{\tilde{R}} = \tilde{C}(0)[\tilde{x}(0)]^{-\alpha/\eta}\{(1-\alpha)t+[\tilde{x}(0)]^{1-\alpha}\}^{(\alpha-\eta)/(1-\alpha)\eta}e^{-(\delta/\eta)t}. \qquad \text{(A3.10)}$$

We may then use the Special Case assumption (3.20) that $\eta=\alpha$ (and since both $\eta<\alpha$ and $\eta>\alpha$ are allowed by the parameter restrictions, there is no reason to suppose that $\eta=\alpha$ will produce atypical results) to give

$$\dot{\tilde{R}} = [\tilde{C}(0)/\tilde{x}(0)]e^{-(\delta/\alpha)t}. \qquad \text{(A3.11)}$$

2. From (A3.4), the Hotelling rule with Cobb-Douglas production is $\dot{x}/x=\dot{K}/K-\dot{R}/R=F/K$, whence $\dot{R}=R(\dot{K}-F)/K=-C/x$. The exponent of $\{(1-\alpha)t+[\tilde{x}(0)]^{1-\alpha}\}$ in (A3.10) is incorrectly given as α/η in DH74 (p17).

This can be integrated, using the initial and terminal conditions $\widetilde{S}(0)=S_0$ and $\lim_{t\to\infty}\widetilde{S}(t)=0$[3], to give the PV-optimal paths of the resource stock and flow:

$$\widetilde{S}(t) = S_0 e^{-(\delta/\alpha)t}, \quad \widetilde{R}(t) = (\delta/\alpha)S_0 e^{-(\delta/\alpha)t} \tag{A3.12}$$

which with (A3.8), (A3.9) and $\widetilde{K}(0)=K_0$ give the PV-optimal paths of consumption and capital:

$$\widetilde{C}(t) = [\widetilde{C}(0)/\widetilde{x}(0)]\{(1-\alpha)t+[\widetilde{x}(0)]^{1-\alpha}\}^{1/(1-\alpha)}e^{-(\delta/\alpha)t} \tag{A3.13}$$

$$\widetilde{K}(t) = (\alpha/\delta)\widetilde{C}(t) \tag{A3.14}$$

where $\widetilde{C}(0) = \delta K_0/\alpha$; $\widetilde{x}(0) = \alpha K_0/\delta S_0$; $\widetilde{C}(0)/\widetilde{x}(0) = \delta^2 S_0/\alpha^2$ (A3.15).

From Solow (1974, p39), the maximin consumption at time zero is

$$C_0^m = \alpha\{K_0^{2\alpha-1}[(2\alpha-1)S_0]^{1-\alpha}\}^{1/\alpha} \tag{A3.16}$$

Initial sustainability ($\widetilde{C}(0)\leq C_0^m$, which also implies single peakedness) occurs if $\delta\leq\alpha^2[(2\alpha-1)S_0/K_0]^{(1-\alpha)/\alpha}$. ||

In fact, in the Special Case one can prove algebraically that $\widetilde{U}>\widetilde{U}^m$ and $\widetilde{\pi}.\dot{\widetilde{\Sigma}}>0$ can coexist. The current maximin utility $U(\widetilde{C}_t^m)$ comes from (A3.16), by replacing K_0 and S_0 by the current capital and resource stocks $\widetilde{K}$ and $\widetilde{S}$. Aggregate investment $\widetilde{\pi}.\dot{\widetilde{\Sigma}}$ equals $\widetilde{U}_C(\dot{\widetilde{K}}-\widetilde{F}_R\widetilde{R})$; and from (A3.7), $\widetilde{U}_C=\widetilde{C}^{-\eta}$ and $\widetilde{F}_R\widetilde{R}=(1-\alpha)\widetilde{K}^\alpha\widetilde{R}^{1-\alpha}$. One can also calculate net national welfare $\widetilde{Z}=\widetilde{U}+\widetilde{\pi}(t).\dot{\widetilde{\Sigma}}(t)$ from the above formulae for $\widetilde{R}$, $\widetilde{C}$ and $\widetilde{K}$, and check numerically that $\widetilde{Z}>\widetilde{U}_t^m$ always (as in Figure 3.3, which was

3. Defining π_S as the costate variable of the resource stock, this stems from the transversality condition $\lim|_{t\to\infty}\pi_S(t)S(t) = 0$, the adjoint equation $\dot{\pi}_S-\delta\pi_S = -\partial H/\partial S = 0$, where H is the undiscounted Hamiltonian of the PV-maximisation problem, and the condition $\pi_S > 0$ for some t. These conditions all apply to any efficient resource depletion path in a Dasgupta-Heal model, and are implicit in the optimal control analysis on p10 of DH74.

computed for the Special Case with $S_0=K_0=1$, $\alpha=\eta=0.7$ and $\delta=0.1$).

Appendix 3.3 Proof of Lemmas 3.3-3.4

LEMMA 3.3: *The opsustimal consumption path cannot always rise.*

PROOF:

At any point on the opsustimal path, either the sustainedness constraint (3.21) is binding ($\dot{C}=0$) or it is not ($\dot{C}>0$). On an always rising path ($\dot{C}>0$), the constraint is never binding, so the path always satisfies the PV-optimal condition $\dot{C}/C=(F_K-\delta)/\eta(C)$. But then by the proof of Lemma 3.2, consumption is zero asymptotically, a contradiction. ||

LEMMA 3.4: *On a continuous opsustimal consumption path, a period with $\dot{C} = 0$ cannot be followed by a period with $\dot{C} > 0$.*

PROOF:

Assume Lemma 3.4 is wrong, so that the opsustimal path (call it $C^{\#}$) does exist with $\dot{C}^{\#}=0$ in some Period 1 ($t_1 \leq t \leq t_2$) followed by $\dot{C}^{\#}>0$ in some Period 2 ($t_2 \leq t \leq t_3$). (3.19) can be rewritten as

$$F_K = \delta + \eta(C)\dot{C}/C$$

so during a locally PV-optimal period such as Period 2, $F_K U_C$, the marginal PV benefit from saving a small unit of consumption for a small unit of time, equals the PV cost of pure impatience (δU_C) plus the PV cost of declining marginal utility $\eta(C)U_C\dot{C}/C$. So then $F_K>\delta$ in Period 2, and by Lemma 3.1, $F_K>\delta$ during Period 1 as well. But that means $F_K>\delta+\eta(C)\dot{C}/C$ throughout Period 1, so that saving will always have a positive net PV benefit. By the assumed continuity of the opsustimal path, and the construction of Period 1, saving a small amount at the start of Period 1 (before which consumption is rising) and spending the proceeds at the beginning of Period 2 does not break the sustainedness constraint. Such a perturbation will therefore lead to a sustained path with more PV than $C^{\#}$. Hence $C^{\#}$ is not opsustimal. $\|$

LEMMA 3.5: *The opsustimal consumption path is continuous.*

PROOF:

Consider any path $C^{\#}$ with an upward jump at some time $t_J > 0$, as shown in **Figure 3.6**, where

$$C^{\#}(t_J-) = \overline{C} \quad \text{and} \quad C^{\#}(t_J+) = \overline{C} + J_1 > \overline{C}$$

($C^{\#}(t_J-)$ denotes the limit of $C^{\#}(t)$ as t approaches t_J from below, etc; and the case $J_1 < 0$ need not be considered because it breaks the sustainedness condition.) Consider the perturbation to consumption shown by the dotted lines in Figure 3.6, where $0 < \omega \ll 1$. The total saving $\mu\omega^2$ after t_J equals the dissaving $\dot{C}^{\#}\omega^2$ before t_J; plus what $\dot{C}^{\#}\omega^2$, had it been added to capital, would have produced after t_J, which is $O(\omega^3)$. So the net PV of the perturbation is then

$$\dot{C}^{\#}\omega^2[U_C(\overline{C} - \dot{C}^{\#}\omega)] - [\dot{C}^{\#}\omega^2 + O(\omega^3)][U_C(\overline{C} + J_1)](1 - \delta\omega)$$
$$\propto U_C(\overline{C}) - U_C(\overline{C}+J_1) - O(\omega)$$

which is sure to be positive for small enough ω, since $U_C(\overline{C}) > U_C(\overline{C}+J_1)$ by the strict concavity in (3.12). The perturbation thus maintains sustainedness and adds PV to $C^{\#}$, so $C^{\#}$ cannot be opsustimal. $\|$

A trivial extension of Lemma 3.5 shows that the PV-optimal path is also continuous, as we used already in Proposition 3.3.

Appendix 3.4 Proofs of Lemmas 3.6-3.7 and Proposition 3.8

LEMMA 3.6: *The opsustimal resource flow rate R is continuous.*

PROOF:

This is similar to the proof of Lemma 3.5, and only a sketch is given. If R is discontinuous at some time t, then by the continuous differentiability of F and the continuity of K, $F_R(K,R)$ jumps downwards at t. One can then bring forward a small amount of R from just after t to just before t in a way that increases consumption just before t, without affecting consumption just after t (the decrease in output just after t caused by the reduction in R would be offset by a decrease in investment then, which can be more than made up for by the increase in output just before t). So PV would thus be increased, which means a discontinuity in R cannot be opsustimal. ||

LEMMA 3.7: *Aggregate investment is continuous on the opsustimal path, and zero from $T^\dagger$ onwards. Net national welfare is continuous on the opsustimal path, and equal to utility from $T^\dagger$ onwards.*

PROOF:

$\dot{K}-F_RR = F-C-F_RR = F_K(K,R)K-C$, given constant returns to K and R. By Lemmas 3.5 and 3.6 this is the sum of two continuous functions on the opsustimal path, and thus is continuous itself. U_C is also continuous by (3.12), so $U_C[F_K(K,R)K-C]$ (= aggregate investment in utility terms) approaches zero as t approaches $T^\dagger$ from below, since it is continuous and zero from $T^\dagger$ onwards by Hartwick's Rule. Given the differentiability assumptions about U, net national welfare $Z=U+U_C[F_K(K,R)K-C]$ is also the sum of two continuous functions, and therefore continuous, and equal to U from $T^\dagger$ onwards. ||

PROPOSITION 3.8: *Let the transition time between the rising and constant utility phases on the opsustimal path be $T^\dagger$. Then:*

$$t<T^\dagger \Rightarrow \quad U^\dagger(t) \;<\; U^{m\dagger}(t) \;<\; Z^\dagger(t) \;=\; \delta\Psi^\dagger(t) \;=\; \delta PV_t(U^\dagger)$$

$$t\geq T^\dagger \Rightarrow \quad U^\dagger(t) \;=\; U^{m\dagger}(t) \;=\; Z^\dagger(t) \;=\; \delta\Psi^\dagger(t) \;=\; \delta PV_t(U^\dagger)$$

and all five functions are continuous at $t=T^\dagger$.

PROOF:

Proof for the rising phase when $t<T^\dagger$:

Since we know from Proposition 3.7 that utility will be rising and then constant, $U^\dagger(t) < U_t^{m\dagger}$. (Otherwise $U_t^{m\dagger}$ is inefficient, a contradiction.)

Next, since the non-declining utility constraint does not bite during the rising phase of the opsustimal path, the path is *locally* competitive during this phase, in the sense of Asheim (1994, p259). Hence (3.5) holds, as long as the upper limit of integration T is less than or equal to $T^\dagger$. So let $T=T^\dagger$, and we then have:

$$e^{-\delta t}Z^\dagger(t)-e^{-\delta T\dagger}Z^\dagger(T^\dagger) \;=\; \delta\int_t^{T\dagger} U^\dagger(s)e^{-\delta s}ds, \quad \text{which with Lemma 3.7 gives}$$

$$e^{-\delta t}Z^\dagger(t) \;-\; e^{-\delta T\dagger}U^\dagger(T^\dagger) \;+\; \delta\int_{T\dagger}^{\infty} U^\dagger(s)e^{-\delta s}ds \;=\; \delta e^{-\delta t}\int_t^{\infty} U^\dagger(s)e^{-\delta(s-t)}ds$$

$$\Rightarrow \qquad Z^\dagger(t) \;=\; \delta[\mathrm{PV}_t(U^\dagger)] \quad \text{as required.}$$

Next, from the derivative of $Z^\dagger$ in (3.4), the definition and derivative of $\Psi^\dagger$ in (3.7) and (3.8), and Lemma 3.7, we have that $Z^\dagger$ and $\Psi^\dagger$ are continuous functions with the same value at $t=0$ and the same derivative till $T^\dagger$, and so are equal throughout the phase. Finally, since the opsustimal path from $t < T^\dagger$ is not constant, and by standard arguments is unique, the current maximin cannot attain the PV of the opsustimal path, i.e. $U^{m\dagger}/\delta<\mathrm{PV}(U^\dagger)$.

Proof for the constant phase when $t \geq T^{\dagger}$:

(i) $U^{\dagger}$ constant and PV-optimal $\Rightarrow U^{\dagger}$ is efficient, and so equal to $U^{m\dagger}$.

(ii) $U^{\dagger}$ constant $\Rightarrow$ Hartwick's Rule holds, i.e. $\dot{K}^{\dagger}(t) - F_R^{\dagger}(t)R^{\dagger}(t) = 0$
$\Rightarrow Z^{\dagger}(t) = U^{\dagger} + U_C(\dot{K}^{\dagger} - F_R^{\dagger}R^{\dagger}) = U^{\dagger}$.

(iii) $U^{\dagger}$ constant $\Rightarrow \delta\mathrm{PV}_t(U^{\dagger}) = \delta U^{\dagger}\int_t^{\infty} e^{-\delta(s-t)}ds = U^{\dagger}$.

(iv) Because $\dot{K}^{\dagger} - F_R^{\dagger}R^{\dagger} = 0$, $\dot{\Psi}^{\dagger} = 0$, and $\Psi^{\dagger}$ continuous for all t
$\Rightarrow \Psi^{\dagger}(t) = \Psi^{\dagger}(T^{\dagger-}) = Z^{\dagger}(T^{\dagger-}) = Z^{\dagger}(t)$. ||

Appendix 3.5 Proof of Proposition 3.9

Consider **Figure 3.7**, where C_0^m is the maximin level of consumption from time zero. The first part of the proof starts from the given condition

$$(2\alpha - 1)^{1/\alpha}(S_0/K_0)^{(1-\alpha)/\alpha}/\delta > 1 \qquad \text{(A3.17)}$$

The dotted line $A_0B_0D_0$ is a consumption-only perturbation (i.e. one that assumes no change in resource flow, the other control variable) that leads from an initial consumption level reduced by an amount $\mu\omega$ below the maximin path, where $0 < 1 - \mu \ll 1$ and $0 < \omega \ll 1$, to a consumption level maintained permanently at $(1-\mu)\omega$ above the maximin path, starting after a short time ω. Average perturbed consumption is $(\mu - ½)\omega$ lower than the maximin path during $0 \leq t \leq \omega$. The increase in the capital stock at $t = \omega$ is thus $(\mu - ½)\omega^2$, which gives a rise in the permanent consumption level from time ω onwards of

$$(\mu - ½)\omega^2(\partial C_0^m/\partial K_0) = (\mu - ½)\omega^2(2\alpha - 1)^{1/\alpha}(S_0/K_0)^{(1-\alpha)/\alpha} \quad \text{from (A3.16)}$$
$$= (1-\mu)\omega \quad \text{from Figure 3.7.} \qquad \text{(A3.18)}$$

The net gain in PV on the perturbed path is thus

$$-\ (\mu - ½)\omega^2 U_C(C_0^m) + [(1-\mu)\omega U_C(C_0^m)](1 - \delta\omega)/\delta + \mathrm{O}(\omega)$$
$$\propto\ -\ 1 + (2\alpha - 1)^{1/\alpha}(S_0/K_0)^{(1-\alpha)/\alpha}(1 - \delta\omega)/\delta + \mathrm{O}(\omega)$$

from (A3.18). This last expression is positive for small enough ω, thanks to (A3.17). The perturbed path is thus sustained and has more PV than the maximin path, so the latter cannot be the opsustimal path.

The second part starts from the given condition

$$(2\alpha-1)^{1/\alpha}(S_0/K_0)^{(1-\alpha)/\alpha}/\delta \leq 1 \qquad \text{(A3.19)}$$

Assume, contrary to Proposition 3.9, that the opsustimal path $C^{\#}(t)$ is distinct from the maximin path. By Proposition 3.7, it must therefore be like $A^{\#}B^{\#}D^{\#}$ in Figure 3.7, comprising a rising phase up to consumption $C^m_{\#} > C^m_0$ at time $T^{\#}$, and constant consumption equal to $C^m_{\#}$ thereafter. (A3.16) together with $C^m_{\#} > C^m_0$ then gives

$$[S^{\#}(T^{\#})]^{1-\alpha}[K^{\#}(T^{\#})]^{2\alpha-1} > S_0^{\,1-\alpha}K_0^{\,2\alpha-1}, \qquad \text{(A3.20)}$$

and (A3.3) ($R^{\#}(t) > 0$ for all t)

$\Rightarrow \quad S^{\#}(T^{\#}) < S_0,$ which with (A3.20) (A3.21)

$\Rightarrow \quad K^{\#}(T^{\#}) > K_0,$ which with (A3.21)

$\Rightarrow \quad S^{\#}(T^{\#})/K^{\#}(T^{\#}) < S_0/K_0.$ (A3.22)

Now consider the consumption-only perturbation $A^{\#}L^{\#}M^{\#}N^{\#}$. A similar marginal analysis as above shows this perturbation has a net PV gain if

$$-1 + (2\alpha-1)^{1/\alpha}[S^{\#}(T^{\#})/K^{\#}(T^{\#})]^{(1-\alpha)/\alpha}/\delta < 0,$$

which is true from (A3.19) and (A3.22). Thus $A^{\#}L^{\#}M^{\#}N^{\#}$ is sustained and has higher PV than $A^{\#}B^{\#}D^{\#}$, so $A^{\#}B^{\#}D^{\#}$ is not the opsustimal path. ||

Figure 3.1 Illustration of the main issues about PV-optimality and sustainability

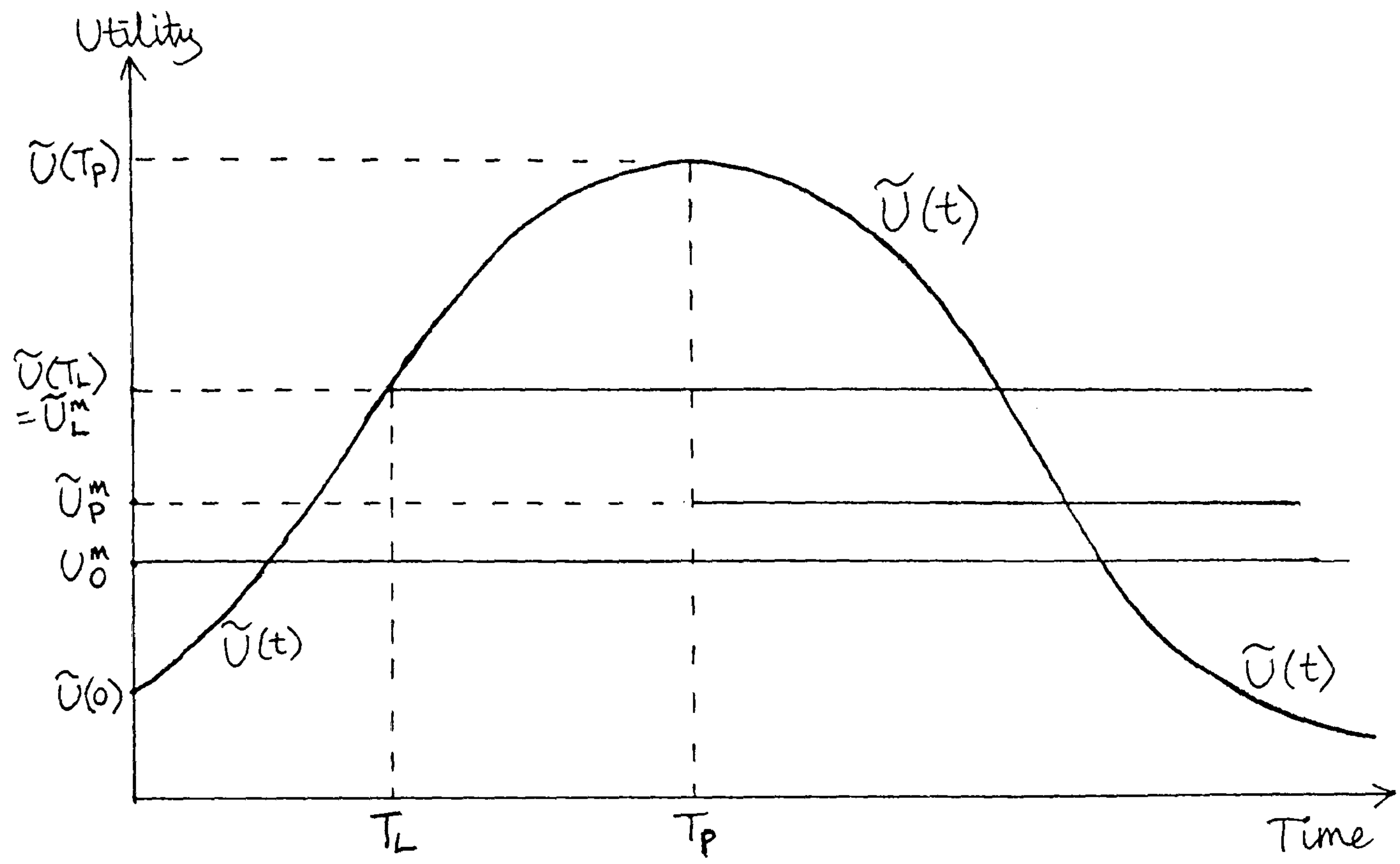


Figure 3.2 Utility, current maximin utility and net national welfare on a PV-optimal path

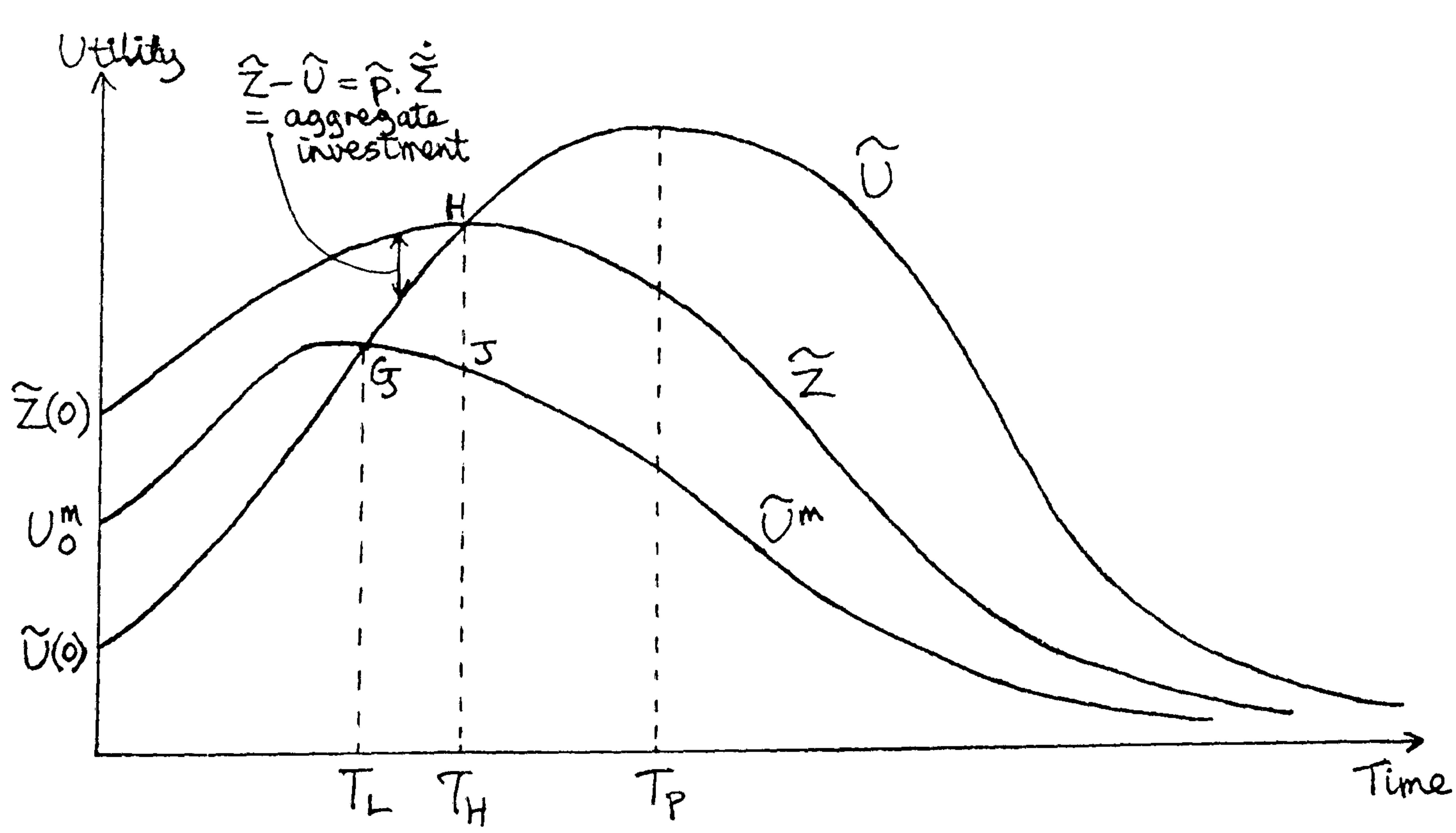


Figure 3.3 Utility, current maximin utility and net national welfare on a PV-optimal path interrupted by an unanticipated constant utility policy

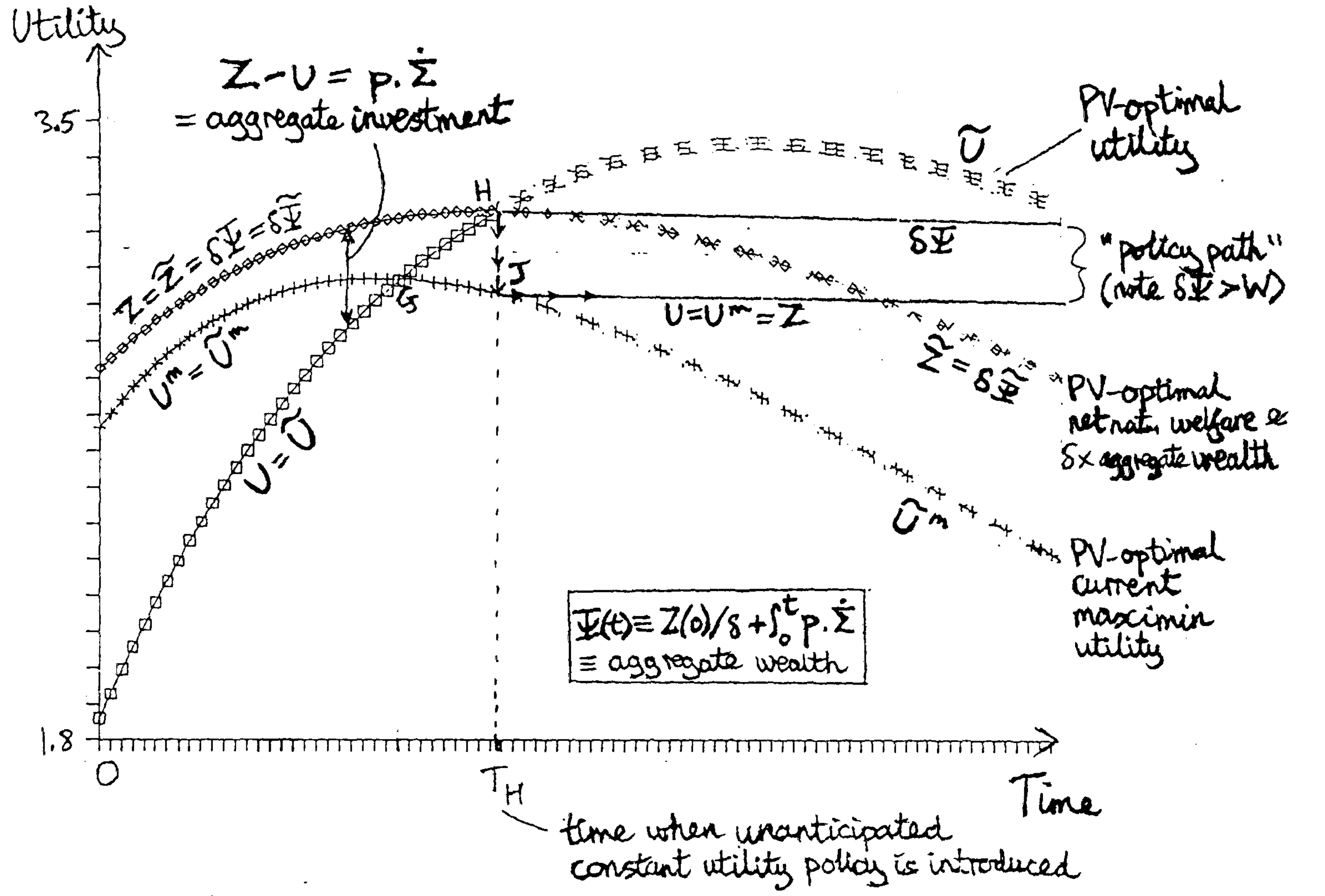


Figure 3.4 Utility, current maximin utility and net national welfare on the opsustimal path

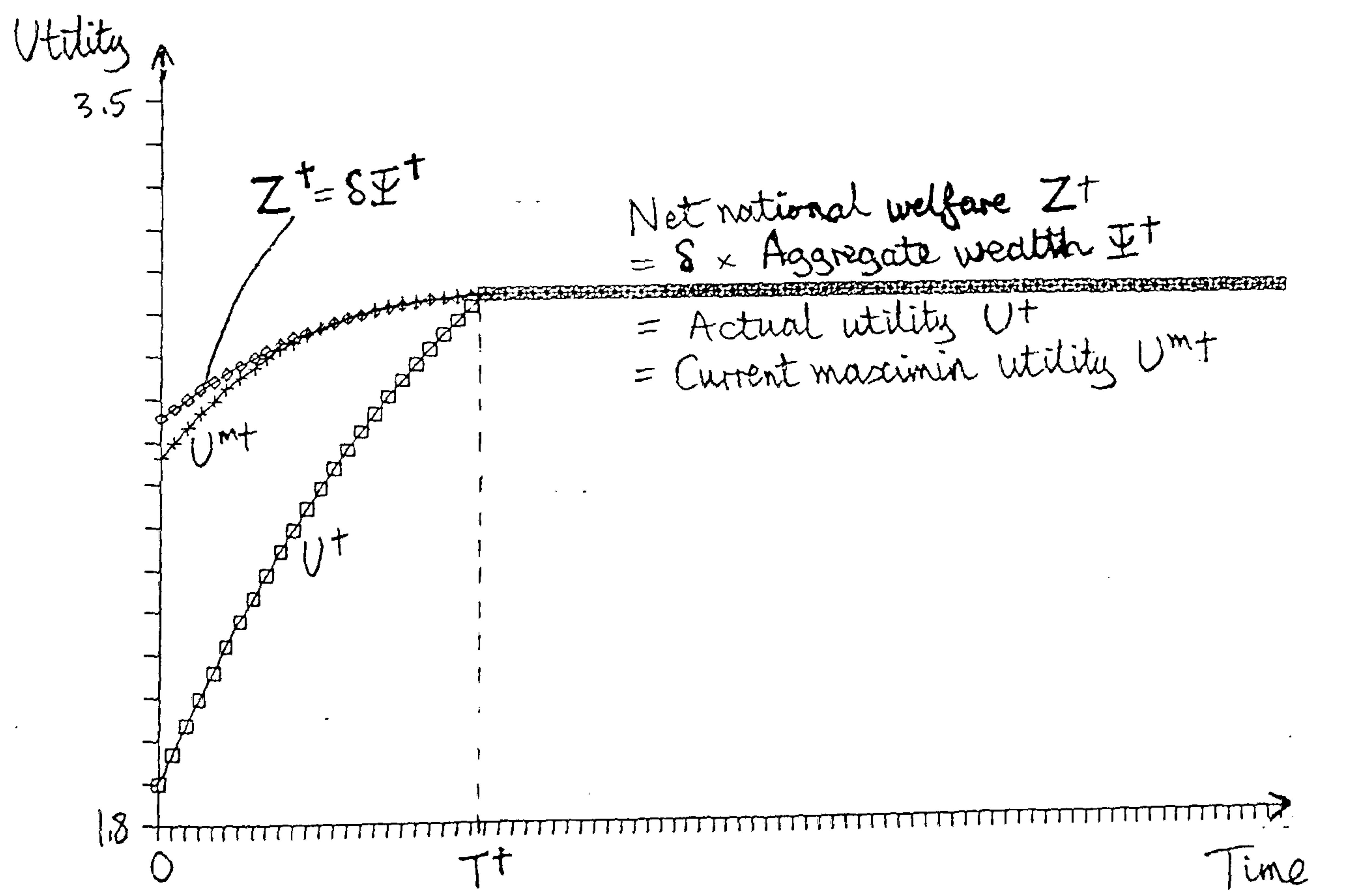
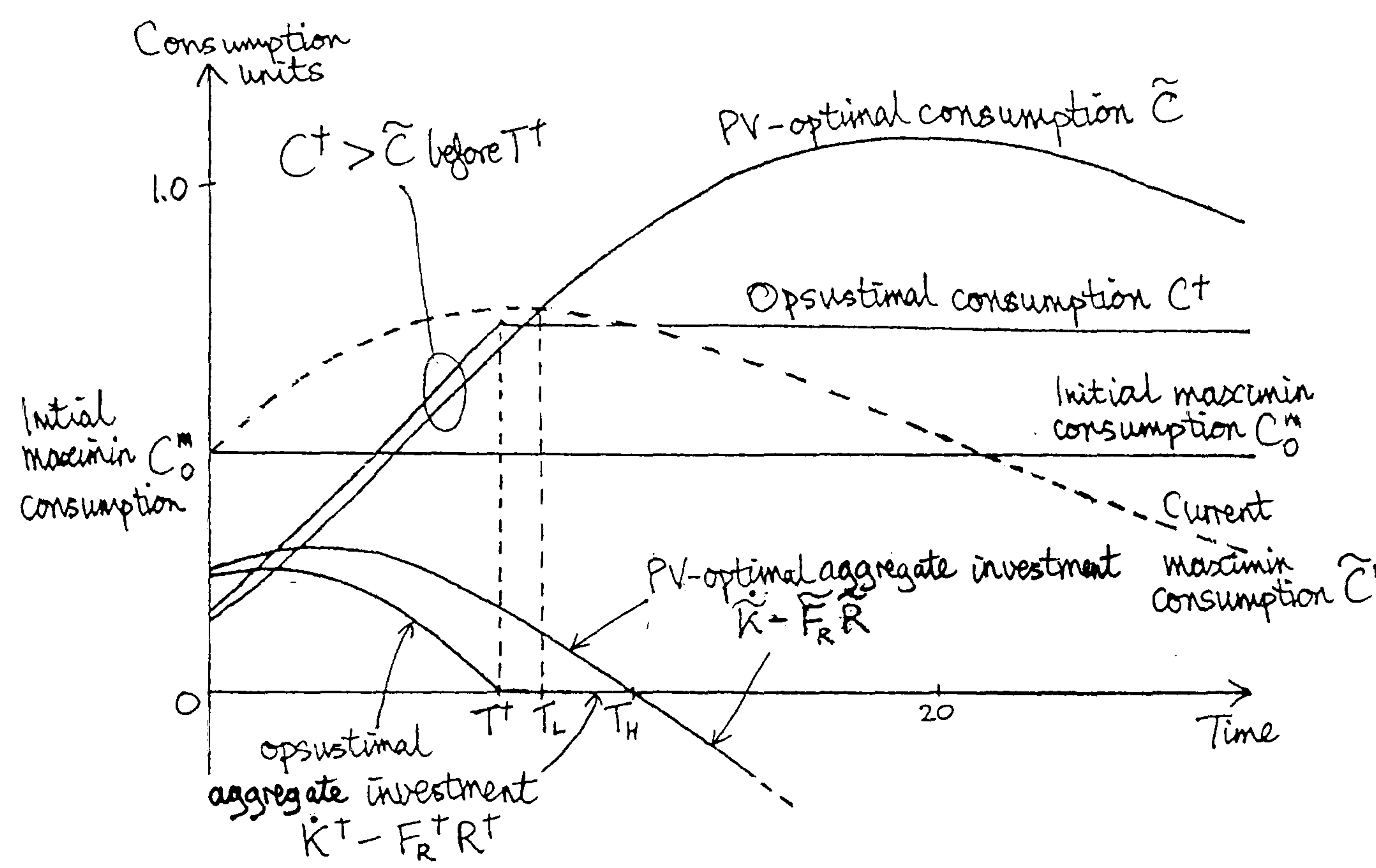
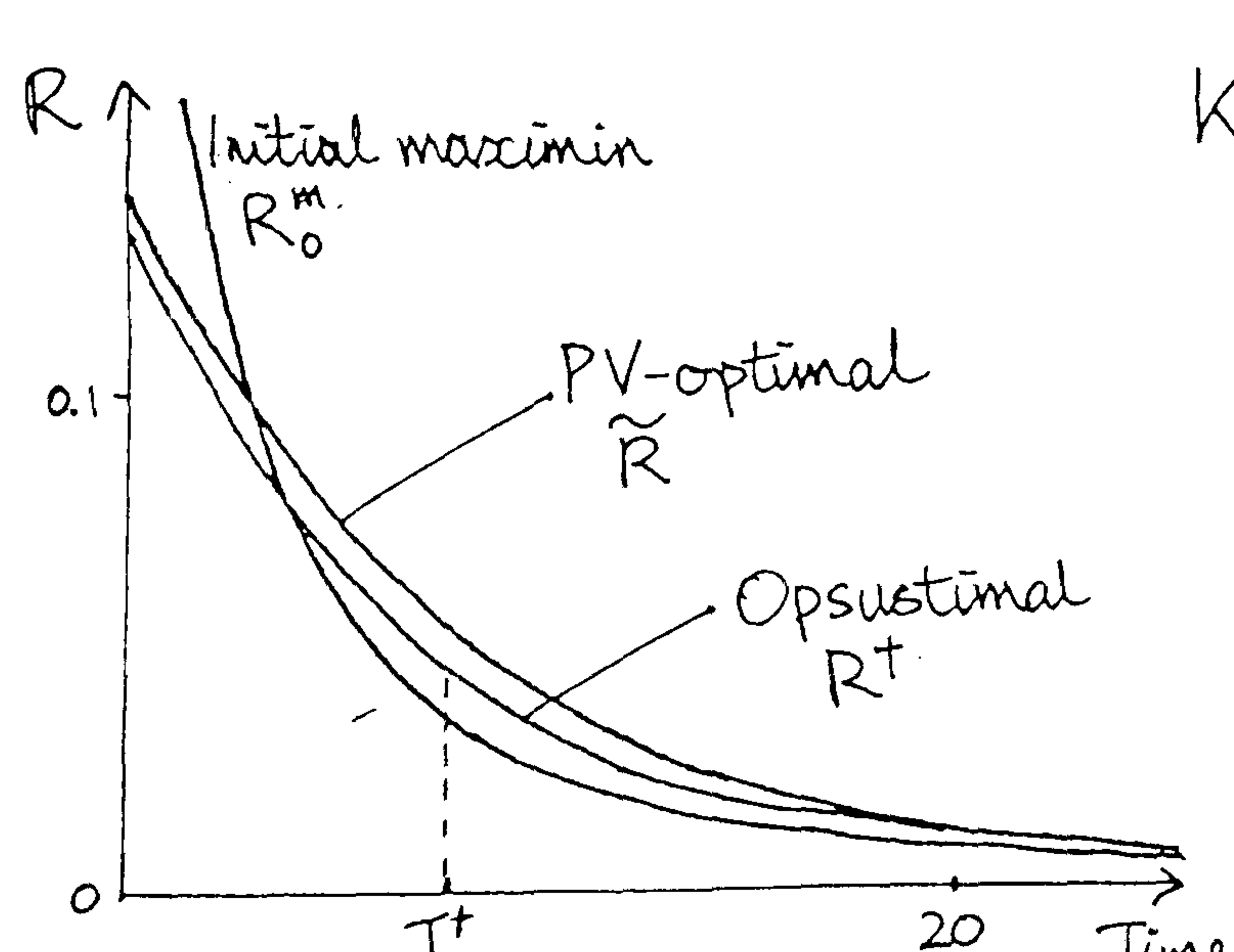


Figure 3.5 Numerical solutions for PV-optimal and opsustimal paths in the Special Case with $\alpha=0.7$, $\delta=0.1$ and $K_0=S_0=1$

(a) Consumption and total investment



(b) Resource flow
(reduced scale)



(c) Capital stock
(reduced scale))

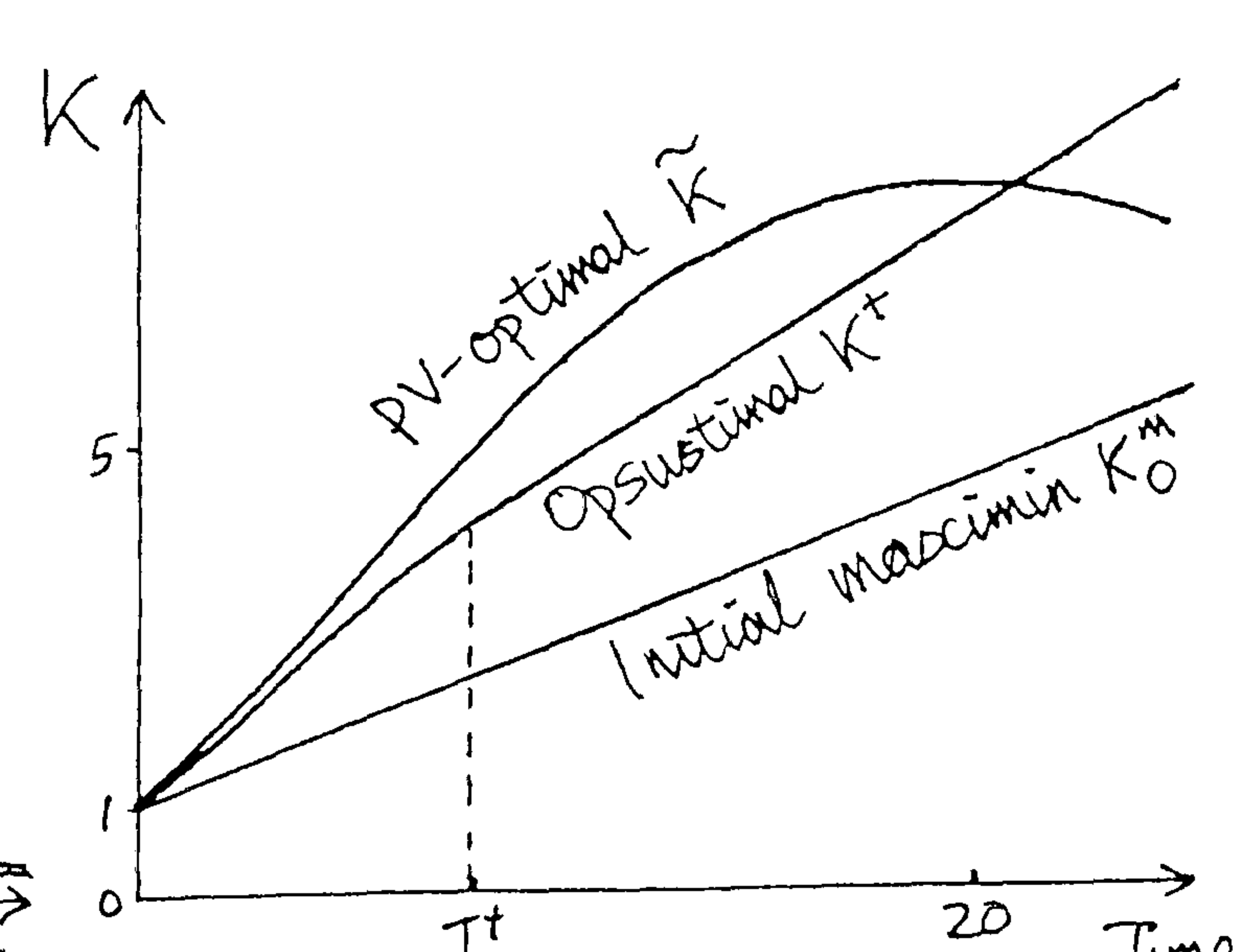


Figure 3.6 Proof of continuity of opsustimal path (for Lemma 4.5)

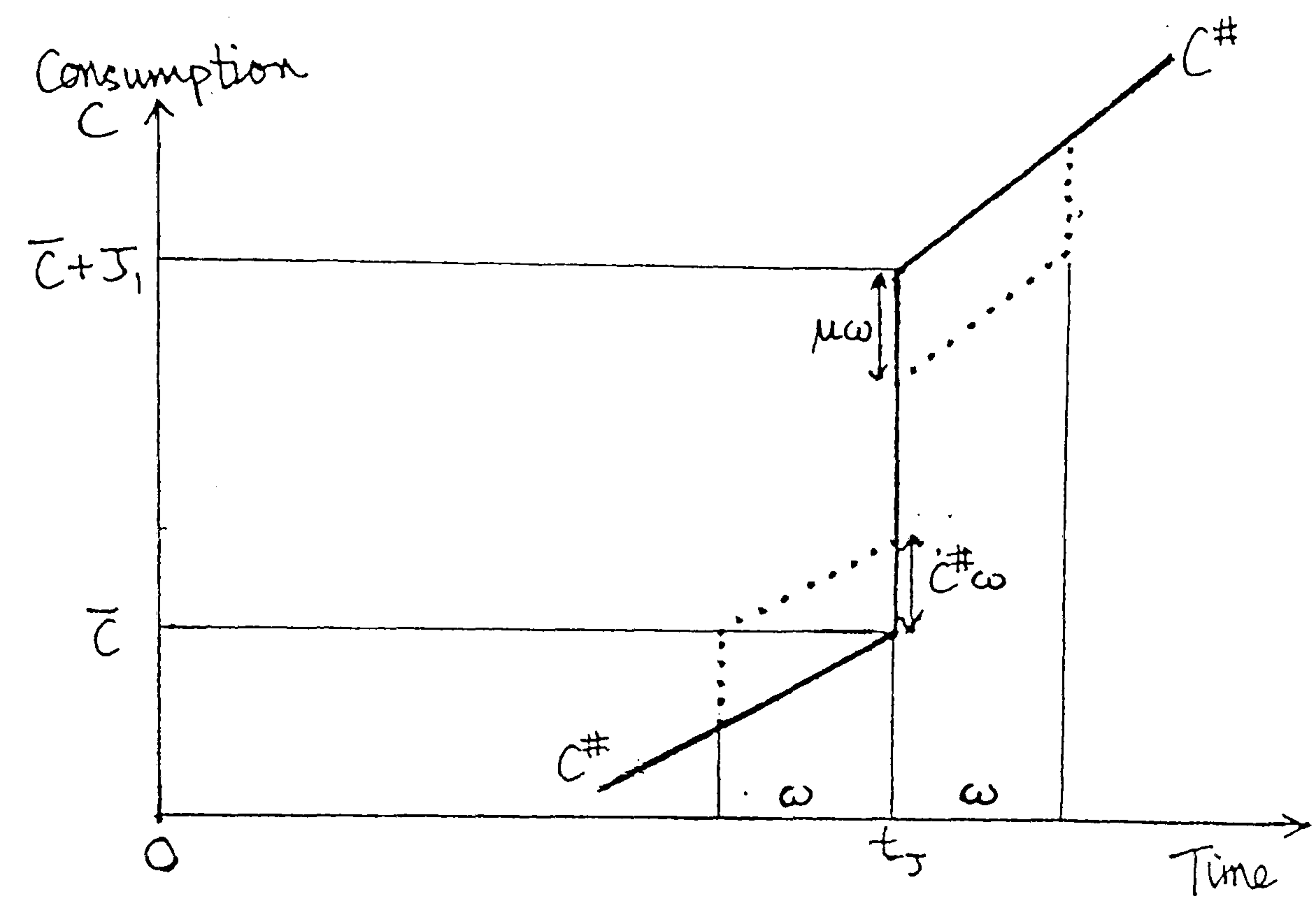


Figure 3.7 Can the opsustimal path improve on the initial maximin? (for Proposition 4.9)

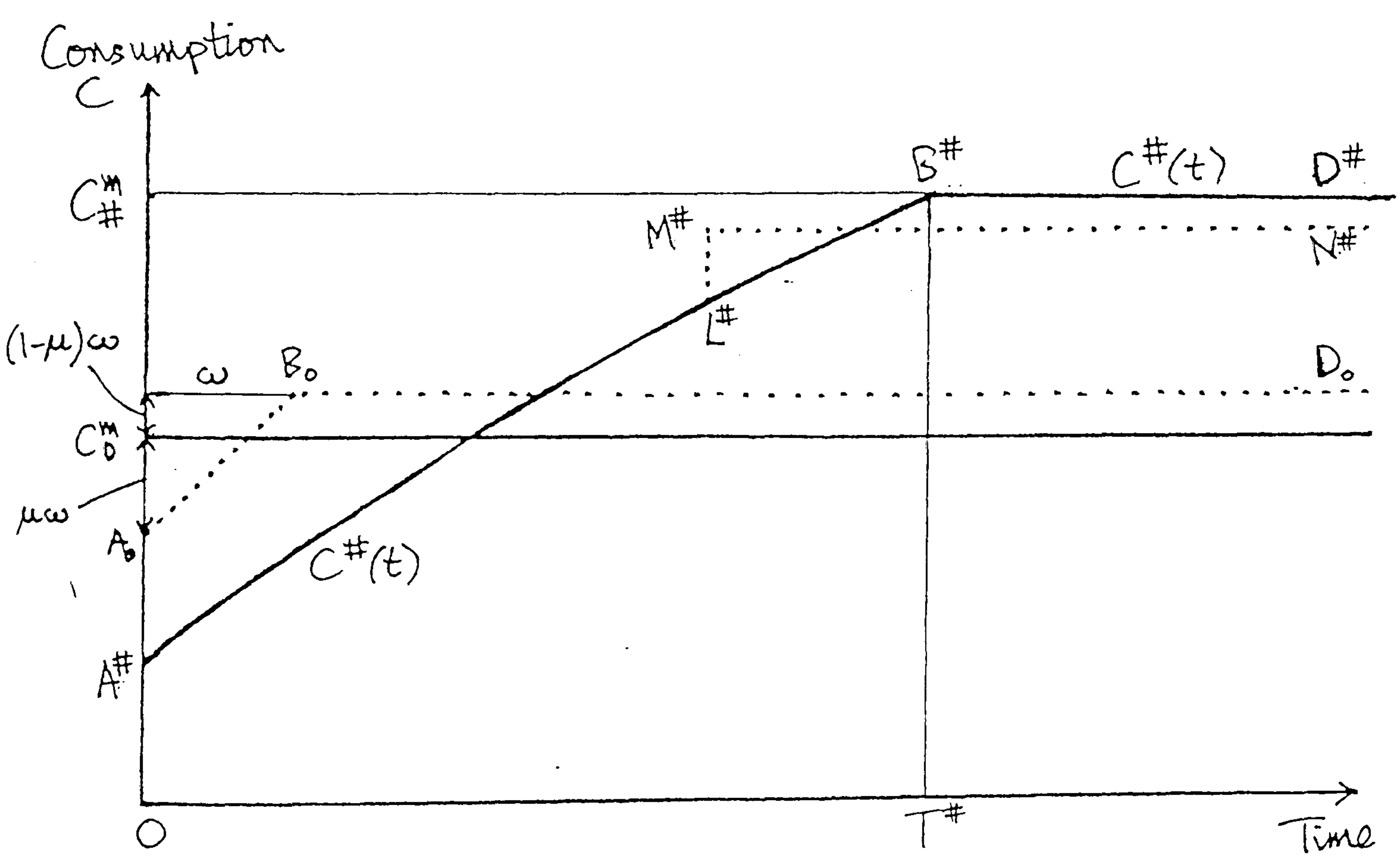
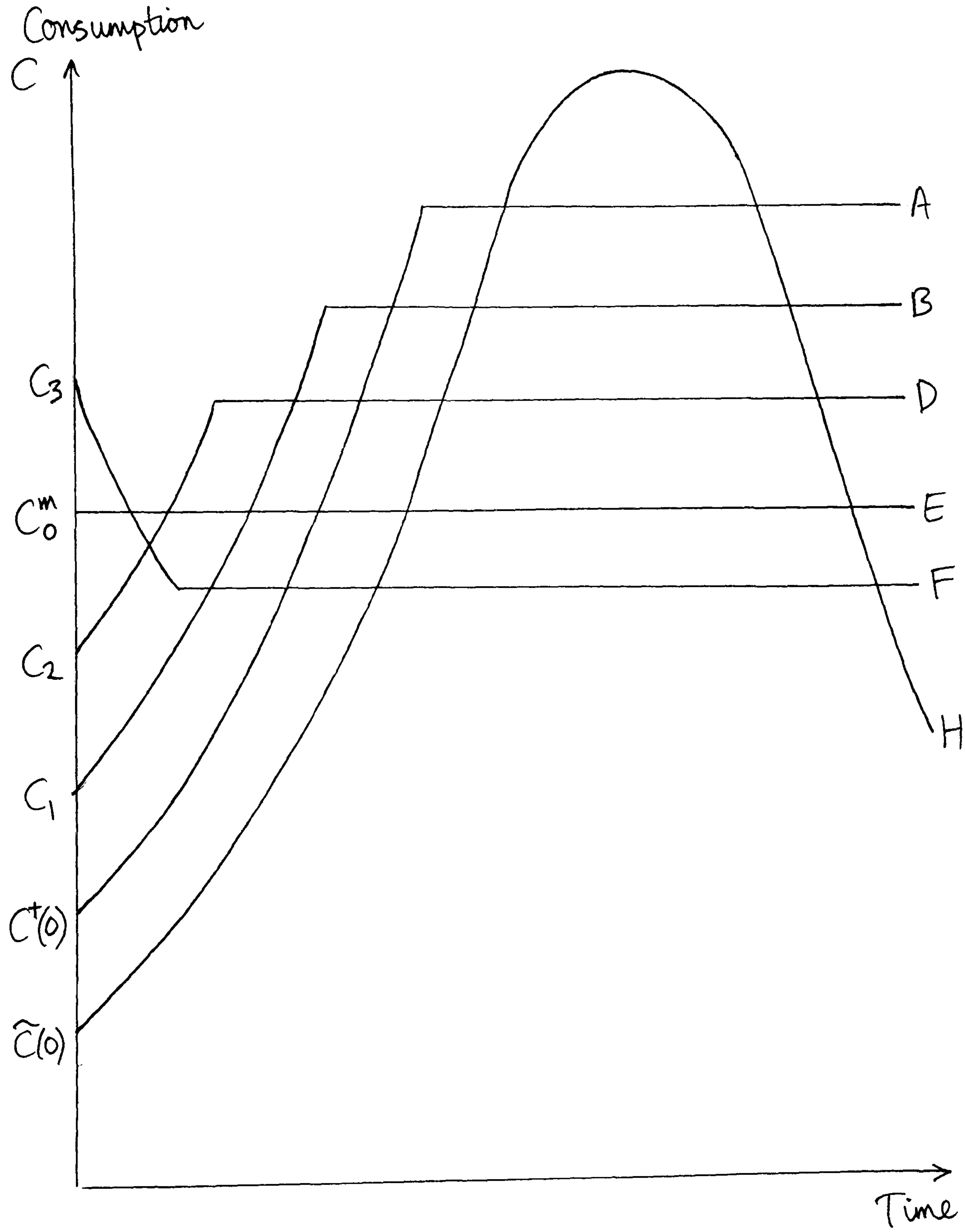


Figure 3.8 Effect of historically-given initial consumption level on opsustimal path



CHAPTER 4

CONCERN FOR SUSTAINABLE DEVELOPMENT IN A SEXUAL WORLD*

4.1 INTRODUCTION

An important if fairly obvious result of Chapter 2, which explored the relationship between controlling environmental deterioration and achieving sustainable development (SD), was that achieving SD may apparently entail a loss of present value of utility (PV) for everyone in the economy. How then can SD policies be justified in terms of individual preferences? The question is important, if such policies are to be democratically implementable (Kennedy 1994). The answer suggested by Conjecture 3.1 was that each agent values the prospect of forever on some scale which is commensurate with PV. However, because of the social amenity value of the common environment, it is inefficient or even impossible for her to achieve SD by individual action, so she finds it individually rational to support a government which pursues an SD policy.

But how can one so justify an SD policy in economies with *no environmental externalities*, as in the models of Chapter 3? Moreover, what if one also assumes constant returns to scale, so that the ownership of capital and resource stocks can then be regarded as being divided equally among

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a large number of identical, price-taking agents? In such circumstances, if individual agents seek SD, why cannot they simply change their private resource depletion and consumption plans accordingly? Why would they wish instead to vote for government which pursues the SD policies analysed in Chapter 3? These are the central questions to be addressed here. And their interest is not merely theoretical. Many people are not persuaded that society-wide, cumulative environmental problems are severe enough in themselves significantly to affect future social utility. To convince them, we would have to find another explanation.

Such sceptics are not very likely either to accept the simple assumption that each agent is concerned to achieve *collective* SD, i.e. NDU forever for an agent representative of the economy as a whole, rather than just NDU forever for herself and her immediate descendants. We will henceforth refer to NDU forever for the *individual* as *sustainedness*, since sustainable 'development' has inevitable connotations of something achieved at a societal rather than an individual level. (As observed in Section 1.2 of Chapter 1, at the level of the whole economy, NDU forever, SD and sustainedness are all formally equivalent concepts in this thesis.) Individual concern for sustainedness could justify SD policy, since collective SD is certainly impossible to achieve by individual action. While not investigating SD as such, Howarth and Norgaard (1993, p347) develop this theme by assuming that "an individual's concern for the welfare of future generations ... may extend to *all* members of future generations", following Marglin's (1963) precedent. They defend this assumption by observing (p352) that:

> "A great many individuals who neither have nor plan to have children profess a concern for future generations, and participants in debates over such diverse issues as the environment, the national debt, and education often argue that members of the present generation are obligated to provide for future generations *in general*, not simply for their own lineal descendants."

While by no means disagreeing with such sentiments, this is not the assumption I choose to make in this chapter, since it seems unnecessarily strong. All I need is the first part of another observation which they do not actually develop as a formal model:

> "In reality, most people live in households where *assets are shared* between members... Parents's efforts to benefit their offspring will thus benefit their offspring's spouses as well, much to the satisfaction of the spouses' parents. Given such interconnections between families, the welfare of children becomes a public good; acting individually, parents will underinvest in their children's futures relative to the outcome that could be achieved through collective action." (Howarth and Norgaard 1993, p351; italics added)

What this recognises is that the asexual agents who populate the vast majority of models of economic growth and development, be they infinitely-lived, or finitely-lived with a single, unbranching line of descendants, do not exist in reality. Sex and marriage exist, and for good genetic reasons (Dawkins 1989, p293), children marry outside the family and *share the use of inherited assets (i.e. bequests)* with their marriage partner in almost all cultures. This sharing has a profound effect on saving and bequests, and therefore on the sustainability of economic growth. To analyse it, we must *abandon the continuous time, infinitely lived person framework* of Chapters 2 and 3. The fact that an infinitely lived person has no offspring and makes no bequests did not matter crucially when we were uninterested in how such bequests might be shared when people die. Here, we are interested, so deaths and births do matter, and we must recognise the existence of separate generations formally within the model.

In doing so, we will follow the lead of Nerlove, Razin and Sadka (1984). They combined parental concern for immediate offspring[1] with marriage

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1. In fact Nerlove, Razin and Sadka assume parents derive utility from their children's *combined inheritances*, rather than from their children's welfare as such.

outside the family. They assumed that:

(1) parents make equal bequests of assets to their children (whether by imparting knowledge, accumulating physical capital or just leaving undepleted natural resources), and in so doing decrease their own hedonistic pleasures and increase their children's;

(2) marriage partners are chosen for 'love' (that is, for reasons unrelated to bequests);

(3) partners share equally any bequests received from their parents, and average out any preferences inherited from their parents; this means that 'individual' in this paper can usually refer equally to one partner or both partners as a couple, without causing ambiguity;

(4) bequests are made before parents know who their children's mates will be;

Given these assumptions, parents bequeath less than the Pareto-optimal amount of resources to their children, because they ignore the benefit that their bequest gives to their children's mates' parents. Thus giving bequests becomes partly a public good, and a matter for public policy.

In particular, the combination of *equal bequests* in (1), *random mating* in (2) and (4), and constant population (which I will assume, even though Nerlove, Razin and Sadka allowed for population growth) will result below in *factors of 2* appearing in expressions for individually optimal bequests where a factor of 1 would be socially PV-optimal. The suboptimality of *laissez-faire* bequests is thus potentially very large for fundamental reproductive reasons, reasons which the kind of sceptics who belittle the significance of suboptimal environmental bequests will find much harder to ignore. True, assumptions (1), (2) and (4) are empirically debatable (and not obviously realistic in the pure resource economy to be modelled below).

In a society where all wealth is passed to one sex of child and/or marriage partners are carefully chosen for their wealth, the suboptimality of family bequests will be much less.[2] But assumptions (1)-(4) are fairly realistic given prevailing modern attitudes towards marriage and sexual equality, and the fact that much of parental bequests is in raising and educating children before they marry. A substantial degree of suboptimality from marriage and bequests is therefore likely, and therefore worth studying in its purest form as a potential justification for SD policy.

In this Chapter I call the Nerlove-Razin-Sadka ignored benefit a *mating-bequest externality*. My contribution is to combine it with an assumption that individual parents are concerned about *family sustainedness*, defined as their children being as well off as they themselves, or the current generation of all parents, are; although I do not provide any detailed empirical or philosophical reasons for assuming this. This concern is included as part of PV in an extended intergenerational welfare function, as suggested in Chapter 2, Section 2.2. I then illustrate, by assuming some particular functional forms in a model of non-overlapping generations, that if parents desire family sustainedness, then it may be a valid goal of public policy, because the mixing of bequests that happens when children mate make it infeasible, or at least inefficient, for parents to achieve sustainedness individually. Further work clearly remains in studying individual concerns for sustainedness, and in building an overlapping generations framework which can also include children's concerns for their parents.

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2. Cole, Mailath and Postlethwaite (1992) analyse the effect of some alternative social norms for mating on bequests and economic growth which differ from assumption (4). And other factors exist, such as automatic bequests of knowledge, which may result *ceteris paribus* in parents bequeathing too much capital and resources rather than too little, and therefore alleviate the sustainedness problem.

A simple game-theoretic version of my model also reveals three less obvious but closely-related conclusions. Firstly, *if* individual sustainedness is feasible, SD as an *overriding* policy concern (i.e. of infinite value, so that SD has to be achieved no matter what the cost in terms of other policy objectives) cannot be derived from individual preferences, because if individuals' concern for sustainedness were unbounded, they would achieve it anyway by individual action. Secondly, if individual concern for sustainedness is too weak, then SD policy is not justified either. Thirdly, feasibility is affected by which form of family sustainedness it is that parents seek. Is it *internal sustainedness*, defined here as children being no worse off than their parents themselves (which is always feasible); or *external sustainedness*, defined here as children being no worse off than their parents' generation as a whole (which is sometimes infeasible)?

We start by considering in Section 4.2 various ways in which overall parental welfare depends on their own hedonistic utility when alive, and on their children's future utility. We then assume one such form of parental concern, the sum of a linear function and a step discontinuity that represents concern for sustainedness, and show its qualititative implications for SD policy. Section 4.3 develops these ideas more precisely using particular functional forms for resource growth and utility, and gives a preliminary game-theoretic analysis of individual sustainedness. Section 4.4 shows how equivalent results to Section 4.3 exist in a reduced-form, asexual version of the model, and argues that collective SD policy can still be justified, even when individual sustainedness action seems *in the asexual model* to be both feasible and as efficient as collective action. Section 4.5 compares the sexual and asexual results in detail, and Section 4.6 concludes.

4.2 PARENTS' CONCERN FOR THEIR CHILDREN

4.2.1 The nature of their concern

We model a world with non-overlapping generations, so that parents live for just one period (indexed by t), and their children live only during period $t+1$ ('period' and 'generation' mean the same length of time throughout).[3] All individuals of the same generation are assumed to be economically identical. Population and the sex ratio are both constant, with each pair of parents producing on average one boy and one girl in the next generation. The concern of each parent in generation t for his or her children is modelled as his or her intergenerational welfare function with paternalistic altruism:

$$W_t = W_t(U_t, U_{t+1}); \quad \partial W_t/\partial U_t > 0, \quad \partial W_t/\partial U_{t+1} > 0; \qquad \text{where}$$

U_t := the hedonistic quality of life that each parent enjoys during period t, which we call *instantaneous utility* or just *utility*, and which will typically depend on current consumption C_t;[4]

U_{t+1} := the utility of each of their children, i.e. the hedonistic quality of life the children enjoy while alive in period $t+1$, which will typically depend on consumption C_{t+1}.

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3. One could instead assume, as Vaughan (1991) and Howarth (1992) do, that their children are alive but with no independent economic role during period t (e.g. living costlessly on 'mother's milk'). But this is mathematically equivalent to, and thus formally no more realistic than, the non-overlapping generations model here.

4. In Chapter 1, the notation for instantaneous utility of a person of age a in an *overlapping* generations model was μ_t^a. However, age is unspecified in the non-overlapping generations framework here, so we may identify μ_t with U_t, the 'net benefit to society in period t'

We could have instead, as Dasgupta (1974) and Asheim (1988) do, called W_t 'utility', and U_t 'felicity', or simply left the latter unnamed. The analysis of non-declining W_t is avoided because it would be more complex than the analysis of non-declining U_t, less in the spirit of what sustainedness means (in my view) to most people, and liable to the sort of dynamic inconsistency problems that both authors identify.

A simple functional form typically assumed for W_t, for analytical convenience as much for as any other reason, is

$$W_t = U_t + U_{t+1}/D; \quad D > 1 \tag{4.1}$$

where D is the intergenerational utility discount factor.[5] This is similar to the seminal formulation of Marglin (1963, p101), except that the latter was in terms of marginal utility. More importantly, Marglin ignored any biological influences on personal preferences by including concerns for all other members of the current generation, and for all other people in the next generation, irrespective of their degree of relatedness to the person holding the preferences. Note also that if instead of (4.1) we had chosen the non-paternalistic, recursive form $W_t = U_t + W_{t+1}/D = \Sigma_0^\infty (1/D)^i U(C_{t+i})$, we would have still have ended up with (4.1) as the maximand for parents in generation t, by applying the derivation in Blanchard and Fischer (1989, p105) to the non-overlapping generations model here. Finally, it is interesting to speculate what value of the discount factor would have been naturally selected in primitive human environments. The obvious guess is that D=2 (equal to about 3% a year over a 25 year generation span), since

5. Perhaps a majority of authors call $1/D$ (<1) rather than D the 'discount factor' (for example Asheim 1988, where $1/D$=b), but I find the definition in (4.1) more memorable since an increase in D then represents increased impatience, as does an increase in the utility discount rate δ in Chapters 2 and 3.

in a constant population each parent can be viewed as bequeathing resources to one child, to whom his degree of relationship is 0.5. However, since such calculations normally apply to transferring resources between parent and child (as in Rogers 1994), rather than transferring utility, some more careful thought may be required here.

The linear welfare function (4.1) needs to be extended if it is to reflect any idea that parents seek sustainedness for their children. **Figure 4.1**, which plots three alternative graphs of $W_t(U_{t+1})|_{Ut=\text{constant}}$, suggests some simple possible extensions. Common to all of them is some kind of 'loss aversion' effect on parents' welfare as children's utility U_{t+1} changes from just above some critical *reference level* U_t^p of utility in the parents' generation (e.g. at points N) to just below it (e.g. at points M). In each case, parents care much more about a fall in their children's utility from N to M, than about a rise of the same magnitude from N to P; so in some sense they place a special value on *sustaining* the utility of the family above the reference level U_t^p as time passes from one generation to the next. But whereas Figure 4.1(a) shows a change of slope of W_t about U_t^p, Figure 4.1(b) shows a discontinuous jump of size B in W_t at U_t^p, with no change in slope, and in Figure 4.1(c) the discontinuity of Figure 4.1(b) is 'smoothed out' so that U_t^p is a point of inflection of W_t rather than a discontinuity.

I suggest that Figure 4.1(c) is something like the true nature of concern that parents feel for their children. This is however pure intuition, and I am not aware of any empirical research which would bear this out (estimating W_t empirically would face the double difficulty of dealing with preferences for intangible utility, and with preferences over long periods of time), or of any philosophical research which studies how such concerns would be consistent with other concerns. I further suggest that a typical reference utility level U_t^p is simply parents' current utility U_t, which is therefore far above the pure survival level for all but the very poorest societies today. This psychological concern for maintaining our children's utility at least as high as our own would, I imagine, have evolved from our biological origin as a subsistence species, when the survival of our genes depended crucially on our children being as well off as ourselves and our contemporaries, and thus able to survive.

For analytical tractability, I will in fact assume the simpler, discontinuous form of concern in Figure 4.1(b). The conclusions stemming from it are probably similar, but more clear-cut, to those stemming from the nonconcave form on Figure 4.1(c). (4.1) thus replaced by

$$W_t = PV_t + BJ\{U_{t+1}, U_t^p\}, \tag{4.2}$$

where

$PV_t = U_t + U_{t+1}/D; \quad D > 1$ is called the 'continuous' or present value (PV) part of welfare;

$B > 0$ is referred to as the benefit or the *value of sustainedness*; and

$J\{a,b\} := 0$ if $0<a<b$; $:= 1$ if $a \geq b > 0$ is a step or 'jump' function.

Two alternatives for the reference level U_t^p are the parents' actual utility U_t, as suggested above, or the average population utility in generation t, say $\bar{U}_t$; for the moment we do not specify which of these is chosen. *Ex ante* such differences are important, as we show in the game-theoretic analysis of Section 4.3.3, but *ex post* there will be no difference between them, as all families are indeed identical. Our *ex post* definition of sustainedness as non-declining utility is thus as in Howarth and Norgaard (1992), and consistent with the definitions already given in Chapter 1:

$$U_{t+1} \geq U_t \quad \text{for all } t.$$

4.2.2 *An overview of externality-correction policies in a sexual world with sustainedness concerns*

The welfare function (4.2) gives rise to two analytically distinct approaches to using government policy to maximise social welfare in a sexual world. Both approaches arise from the mating-bequest externalities caused by parents' concern for their children, who mate outside the family and share their inheritance with their mates. Any increase in bequest that two parents in period t give to their children will then increase the 'warm glow' of altruistic welfare enjoyed by the parents of their children's mates. The increase occurs *continuously* through the term U_{t+1}/D in PV_t, and *discontinuously* through the step function $BJ\{U_{t+1}, U_t^p\}$. But if mating is random and happens after bequests have been made (assumptions which are made throughout this chapter), free markets externalise both these sources of extra intragenerational value of a bequest. Social welfare will then be maximised by government policies, such as a consumption tax, which give an incentive at the margin for higher bequests.[6]

In the first analytical approach to analysing what tax level maximises welfare, we assume that the sustainedness value B is zero. We thus ignore the discontinuous part of the externality in parents' welfare. Recognising that any increase in children's utility U_{t+1} requires a sacrifice of parents' utility U_t, a parent's overall welfare W_t is therefore maximised by setting

$$dPV_t/dU_{t+1} = dU_t(U_{t+1})/dU_{t+1} + 1/D = 0, \qquad (4.3)$$

assuming that $d^2PV_t/dU_{t+1}^2 = d^2U_t/dU_{t+1}^2 < 0$. The solution to (4.3) will

6. The literature on 'Ricardian equivalence', which purports to show that government redistributive policies are ineffective because they are cancelled out by individual actions (see for example Barro 1974 and Bernheim and Bagwell 1988) is irrelevant here, as this literature studies *lump sum* taxes rather than the incentive taxes modelled here.

be called the *zero-B social optimum* for U_{t+1}, denoted by $\widetilde{U}_{t+1}$. The child's utility level that individual parents select as optimal in the *zero-B free market* (where there is no government intervention, no sustainedness value B, and each family assumes that it acts alone), denoted by $U_{t+1}{}^{\mu}$. Because of the continuous mating-bequest externality, $U_{t+1}{}^{\mu}$ is always less than $\widetilde{U}_{t+1}$. One way to maximise W_t for everyone would then be for the government to impose a revenue-neutral consumption tax at the level needed to raise an individual parent's plan for his child's utility from $U_{t+1}{}^{\mu}$ to $\widetilde{U}_{t+1}$.

The second analytical approach is to assume a finite, positive value of sustainedness ($B > 0$). There are then three different ways in which this can interact with the various possible relative positions of $U_{t+1}{}^{\mu}$, $\widetilde{U}_{t+1}$ and the reference level U_t^{ρ} to affect the tax level that is needed to reach a global maximum of overall welfare W_t, as shown in **Figure 4.2**, which assumes that everyone acts together.[7] Firstly, in Figures 4.2(a) and 4.2(b), it happens that $U_t^{\rho} < \widetilde{U}_{t+1}$, so sustainedness concerns have no effect on the collective choice of $\widetilde{U}_{t+1}$ as the globally maximum utility level in period $t+1$. (They might make the sustained point S in Figure 4.2(b) *individually* preferable to the unsustained zero-B free-market solution M, but we will not analyse this case here.) Secondly, Figure 4.2(c) shows the case which will most concern us here. Here $\widetilde{U}_{t+1} < U_t^{\rho}$ and $B > PV_t(\widetilde{U}_{t+1}) - PV_t(U_t^{\rho}-)$ (where the $U_t^{\rho}-$ notation means the value just left of the discontinuity at U_t^{ρ}), so that a policy to move to the zero-B optimum point L is not globally optimal. Instead, an explicit SD policy which achieves $U_{t+1} = U_t^{\rho}$ at point G is globally optimal (ignoring the possibility, which we study in Section 4.3.3, that individuals find it individually rational to reach G by their own

7. Although U_t^{ρ} very probably varies as U_t and therefore U_{t+1} vary, there will nevertheless be a fixed value of U_t^{ρ} at which it equals U_{t+1}.

effort). Thirdly, Figure 4.2(d) shows the case where the zero-B optimal level $\widetilde{U}_{t+1}$ is not sustained, but the cost of sustainedness $PV_t(\widetilde{U}_{t+1}) - PV_t(U_t^\rho -)$ is greater than its value B, so that it is globally optimal to stay at the zero-B optimal point.

We now illustrate these general principles by developing a model with particular functional forms, in which we can calculate the values of U_{t+1}^μ, $\widetilde{U}_{t+1}$ and U_t^ρ, and the nature of the SD policies needed to reach U_t^ρ. We start in Section 4.3.1 by calculating the zero-B results, which ignore specific sustainedness concerns. We then add in these concerns in Section 4.3.2. In Section 4.3.3 we consider the question of individual sustainedness: can collective SD policy ever be redundant, because individual families may be able and motivated to achieve sustainedness by their own initiative?

4.3 A MODEL OF RESOURCE DEPLETION AND BEQUESTS IN A SEXUAL WORLD

4.3.1 The model without sustainedness concerns

This section uses the same framework as the general model in Section 4.2.1, and then assumes a particular functional form for utility. Successive, non-overlapping generations each have the same, large number of economically identical couples of opposite sex, and random mating occurs outside the family to form the couples of each new generation. Consumption (which gives utility) directly depletes a durable resource stock, the remainder of which grows at a constant, exogenous rate between generations.[8] Each couple pass on their remaining resource stock as equal-

8. Whether the 'resource' is instead called a 'commodity' (Dasgupta 1974), 'corn' (Howarth 1992), a 'renewable resource' (Mourmouras 1993) or 'natural capital' (Kennedy 1994) is unimportant. The equations relating consumption, asset growth

sized bequests to their two children in the next generation, who pool their resource inheritances with their mates. At the beginning of period t, a parent is endowed (by a bequest from her own parents) with a stock S_t of resource. Because her mate is economically identical, he receives the same endowment S_t. They each choose to consume a fraction g of this joint stock during the period, so *per capita* consumption is

$$C_t = gS_t. \tag{4.4}$$

The government in period t levies taxes at rate ϕ on parents' consumption. The total tax paid by each parent is ϕgS_t, and the government achieves revenue-neutrality by paying this back as a lump sum of $\phi\overline{g}S_t$, where $\overline{g}$ is the average consumption fraction (henceforth called propensity) across all couples in period t. All couples make the 'zero conjecture' and regard $\overline{g}$, and hence the lump sum, as something their behaviour cannot influence.[9] They then leave whatever stock remains to grow during the period by an exogenous (technical or biological) growth factor Γ (>1).

To find the government's optimal choice of tax rate ϕ, note that each couple's remaining resource in period t, after accounting for consumption, taxes and the lump sum refund, is $2(1-g-\phi g+\phi\overline{g})S_t$, which grows to $2(1-g-\phi g+\phi\overline{g})\Gamma S_t$ at the start of period $t+1$, that is, to a bequest of $(1-g-\phi g+\phi\overline{g})\Gamma S_t$ per child. From the couple's point of view, the resource endowment of either child's (say a son's) household at the start of period $t+1$ will be

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and bequests set out below are the same as in all these models, and form the discrete time equivalent of Special Case 1 of the cake-eating model of Chapter 2, with a zero environmental preference ϵ.

9. See Hirshleifer (1988, Note 17) for comments on consistency problems with this zero conjecture.

$$2S_{t+1} = (1-g-\phi g+\phi \bar{g})\Gamma S_t + X \tag{4.5}$$

where X is his mate's bequest from her parents. Since she is randomly selected, his parents treat X as another parameter beyond their influence. We next assume that:

[A1] *If parents are not concerned about sustainedness or other issues of intergenerational equity, they exert no influence over their children's consumption propensities.*

They therefore assume that their son and his mate will choose g freely when their time comes. In any case this will be a population-wide choice, so we represent the child's family's propensity by another exogenous unknown parameter y. Parents therefore calculate from (4.5) that their son's consumption C_{t+1} is:

$$C_{t+1} = y[(1-g-\phi g+\phi \bar{g})\Gamma S_t + X]/2, \tag{4.6}$$

We now assume a specific form for the utility and welfare functions of all generations:

$$U_t = \log(C_t) \qquad W_t = \log(C_t) + (1/D)\log(C_{t+1});\ D > 1 \tag{4.7}$$

$U_t(C_t)$ is thus strictly concave, as required in Section 4.2.2. Together (4.4), (4.6) and (4.7) give parental welfare as

$$W_t = \log(gS_t) + (1/D)\log\{y[(1-g-\phi g+\phi \bar{g})\Gamma S_t + X]/2\}. \tag{4.8}$$

Parents maximise (4.8) by choosing a consumption propensity g to satisfy:

$$\partial W_t/\partial g = 1/g - (1+\phi)\Gamma S_t/\{D[(1-g-\phi g+\phi \bar{g})\Gamma S_t + X]\} = 0,$$

which treats $\bar{g}$, X and y as parameters ($\partial \bar{g}/\partial g = \partial X/\partial g = \partial y/\partial g = 0$), as noted above. But since all agents in the economy are identical, the child's mate's consumption propensity $\bar{g}$ and bequest X will be the same *ex post* as the child's, i.e. $\overline{g}=g$ (which also means that the government's budget is balanced) and $X=(1-g)\Gamma S_t$. So every parent's privately optimal choice of g, given the government's tax rate ϕ, is then given by

$$1/g = (1+\phi)/\{2D(1-g)\} \quad \Rightarrow \quad g = \hat{g} := \mathbf{2D/(2D+1+\phi)} \tag{4.9}$$

(using the overscript ^ to denote the privately optimal response to government intervention, as in Chapters 2 and 3). The factor of 2 multiplying D in (4.9) (and in many similar results below) is of general significance. It is the 'sexual factor' which reflects the halving of the control that parents have over their children's bequests because their children marry outside the family. (In contrast, the absence of Γ in (4.9) is a particular effect of the logarithmic form of U_t in (4.7).) From (4.6), *assuming that the government in generation t+1 adopts the same tax policy as the government in generation t*, $y=\hat{g}$ *ex post* and period $t+1$ consumption is:

$$C_{t+1} = \hat{g}(1-\hat{g})\Gamma S_t. \tag{4.10}$$

From (4.4), (4.7), (4.9) and (4.10), privately optimal per capita consumption in periods t and $t+1$, and overall welfare in period t, are then

$$\hat{C}_t = 2DS_t/(2D+1+\phi); \quad \hat{C}_{t+1} = 2D(1+\phi)\Gamma S_t/(2D+1+\phi)^2; \tag{4.11}$$

$$\begin{aligned} \hat{W}_t(\phi) &= (1+1/D)\log[2DS_t/(2D+1+\phi)]. \\ &\quad + (1/D)\log[(1+\phi)\Gamma/(2D+1+\phi)]. \end{aligned} \tag{4.12}$$

With no intervention ($\phi = 0$), from (4.9)-(4.12) we end up with the free market equilibrium (denoted by superscript $^{\mu}$ as in Chapter 2):

$$g^\mu = 2D/(2D+1) \tag{4.13}$$

$$C_t^\mu = 2DS_t/(2D+1); \quad C_{t+1}{}^\mu = 2D\Gamma S_t/(2D+1)^2; \tag{4.14}$$

$$W_t^\mu = (1+1/D)\log[2DS_t/(2D+1)] + (1/D)\log[\Gamma/(2D+1)]. \tag{4.15}$$

But from (4.12), it is in fact optimal for the government to intervene, by setting a tax rate given by

$$\partial \hat{W}_t/\partial\phi = 1/[D(1+\phi)] - (1+2/D)/(2D+1+\phi) = 0$$

$$\Rightarrow \phi = \tilde{\phi} := \mathbf{(D-1)/(D+1)} \tag{4.16}$$

(again using the tilde $\tilde{}$ to denote zero-B optimal).[10]

From (4.9)-(4.12) and (4.16), the corresponding zero-B optimal consumption propensity, consumptions per capita and welfare level can be shown to be:

$$\tilde{g} = (D+1)/(D+2); \tag{4.17}$$

$$\tilde{C}_t = (D+1)S_t/(D+2); \quad \tilde{C}_{t+1} = (D+1)\Gamma S_t/(D+2)^2; \tag{4.18}$$

$$\tilde{W}_t = (1+1/D)\log[(D+1)S_t/(D+2)] + (1/D)\log[\Gamma/(D+2)]. \tag{4.19}$$

Comparing these zero-B socially optimal results with the free market results (4.13)-(4.15), it is immediately obvious that $\tilde{g} < g^\mu$ (so intervention has reduced the consumption propensity) and $\tilde{C}_t < C_t^\mu$, and fairly obvious that $\tilde{C}_{t+1} > C_{t+1}{}^\mu$. To show that the government intervention improves welfare (i.e. that $\tilde{W}_t > W_t^\mu$, which we need to do since we did not check any second-order conditions above) takes a few steps of algebra, spelt out in Lemma A4.3 in the Appendix.

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10. Variants of $\tilde{\phi}$ useful for manipulation are $1+\tilde{\phi} = 2D/(D+1)$ and $2D+1+\tilde{\phi} = 2D(D+2)/(D+1)$.

4.3.2 *Adding in sustainedness concerns*

If sustainedness has a positive value ($B>0$), then from (4.2) and (4.7),

$$W_t = \log(C_t) + (1/D)\log(C_{t+1}) + BJ\{U_{t+1}, U_t^p\}; \quad D > 1$$

Ex post sustainedness is defined in Section 4.2.1 above as $U_{t+1} \geq U_t$, which in turn means $C_{t+1} \geq C_t$. We now calculate the parameter values that would make the various equilibria of Section 4.3.1 sustained.

From (4.14) the free market equilibrium is sustained and only if

$$\Gamma \geq \Gamma^{\mu} := 2D+1;$$

and from (4.18), the zero-B social optimum is sustained if and only if

$$\Gamma \geq \tilde{\Gamma} := D+2 \quad (< 2D+1 \text{ since } D>1). \tag{4.20}$$

These two critical values Γ^{μ} and $\tilde{\Gamma}$ of the growth factor divide the policy outcomes into three distinct cases:

Case I.	$\mathbf{\Gamma \geq 2D+1}$	corresponding to Figure 4.2(a);
Case II.	$\mathbf{2D+1 > \Gamma \geq D+2}$	corresponding to Figure 4.2(b);
Case III.	$\mathbf{D+2 > \Gamma}$	corresponding to Figure 4.2(c) or (d).

Assuming that the mating-bequest externality from the continuous part of parents' concern for their children is internalised by a government consumption tax of size $\tilde{\phi} = (D-1)/(D+1)$, only in Case III is the zero-B optimal ($\sim$) equilibrium is unsustained. Since from (4.11) the general policy equilibrium is sustained if and only if

$$\Gamma \geq (2D+1+\phi)/(1+\phi),$$

the required consumption tax to achieve sustainedness for all individuals in the economy (i.e. SD) is

$$\phi = \phi^{ss} := (2D+1-\Gamma)/(\Gamma-1), \tag{4.21}$$

where the superscript ^{ss} denotes 'socially sustained', since the tax is an instrument of social (i.e. government) policy.[11] From (4.9) and (4.21) the resulting sustainable consumption propensity in generation t is

$$g = g^{ss} := (\Gamma-1)/\Gamma. \tag{4.22}$$

This is as expected, since $g^{ss}\Gamma S_t = (\Gamma-1)S_t$, so just achieving SD requires that exactly the surplus growth in the resource is consumed.

Note that by using (4.11) to calculate ϕ^{ss}, we are elevating the assumption made just before (4.10) into something quite important:

[A2] *If the government of generation t adopts an SD policy, we assume that it can commit the governments of all subsequent generations to follow the same policy.*

This assumption, which echoes Solow (1974, p36), contrasts with assumption [A1] in Section 4.3.1 where parents exert no influence over their children's consumption preferences. Here, we have to assume that, acting collectively, parents influence the consumption preferences of all future generations. Without this power of 'infinite future commitment', it would be impossible to calculate what SD policies for all time should be. The power of infinite future commitment was also assumed (although implicitly) in the calculations of SD policies in the infinitely-lived, representative person framework of Chapters 2 and 3.

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11. The model assumes there are no pre-existing, perverse government policies which make the policy equilibrium *less* likely to be sustained than the free market equilibrium would be. This may not be the case in real world applications where perverse incentives are commonplace. Useful variants of (4.21) are $2D/(2D+1+\phi^{ss}) = (\Gamma-1)/\Gamma$ and $(1+\phi^{ss})\Gamma/(2D+1+\phi^{ss}) = 1$.

Achieving SD with the tax ϕ^{ss} will be justified in the sense of maximising overall welfare only if the individual benefit B of SD exceeds its individual cost in terms of PV. We therefore divide Case III into two parts:

Case IIIA $\quad B > PV(\widetilde{U}_{t+1},\widetilde{U}_t) - PV(U_{t+1}^{ss},U_t^{ss})$

Case IIIB $\quad B \leq PV(\widetilde{U}_{t+1},\widetilde{U}_t) - PV(U_{t+1}^{ss},U_t^{ss})$

In Case IIIB, an SD policy is definitely not justified. In Case IIIA, the tax would appear to be justified because it increases overall welfare. But first, as promised, we have explore whether or not *individual* action might ever achieve a sustained equilibrium, thereby rendering collective policy superfluous.

4.3.3 Could individual action make an SD policy redundant?

Could the value B, that each individual family places on its own sustainedness, be high enough for it to be worthwhile for it to seek sustainedness by its own thrift (i.e. the parents decreasing their own consumption level, and bequeathing more to their children), irrespective of what it thinks the rest of society will do? If so, an SD policy, although welfare-improving, will actually be unnecessary in Case IIIA. To answer the question, we first need to return to the remark at the end of Section 4.2.1 that when defining sustainedness, there can be a difference between defining the reference level of utility U_t^{ρ} as individual parents' actual utility U_t or as the average utility of their generation, $\overline{U}_t$. Parents in generation t are defined here as regarding their family line as:

- *internally sustained* if they expect their children to be at least as well off in period $t+1$ as they themselves are in period t (i.e. $U_t^{\rho}=U_t$) ; and o *externally sustained* if they expect their children to be at least as well off in period $t+1$ as the *average* parents in generation t (i.e. $U_t^p = \bar{U}_t$).

Internal sustainedness is always possible, for as parents decrease their consumption towards zero, their own utility declines without limit, while their children's increases; so the two must become equal somewhere, even though the parents may then be much worse off than their contemporaries in generation t. For both internal and external sustainedness we assume in contrast to the assumption [A1] of no parental influence in Section 4.3.1, that:

[A3] *If a particular pair of parents take action to achieve sustainedness just for their own family, they can and do raise each of their children to adopt the same consumption propensity as they (the parents) choose.*

If we did not assume this, the collection of all families acting individually in pursuit of sustainedness (if they are motivated to do so) would not produce the same outcome as the current government following a collective SD policy, given the infinite future commitment implied by assumption [A2]. An immediate implication of [A3] is that external sustainedness may be impossible: no matter how much a particular pair of parents scrimp and save, if their contemporaries are high consumers, their children (who will inherit their parents' low consumption propensity) may never reach this high level of consumption. It is hard to know whether internal or external sustainedness is more likely to be the 'natural' concern for people to have, but here we simply explore the implications of the two definitions.

Our analysis of the policy implications of internal versus external sustainedness for policy starts by assuming that just *one* family is concerned about individual sustainedness in the model of Section 4.3.2. We assume also that governments in all periods are already internalising the continuous mating-bequest externality by taxing consumption at a rate $\tilde{\phi}=(D-1)/(D+1)$ (from (4.16)), so that other couples' consumption propensities are all $\tilde{g}=(D+1)/(D+2)$ from (4.17); and that there is indeed a sustainedness problem, so $\Gamma<D+2$ from (4.20). To achieve **internal sustainedness**, the per capita consumption level that a particular family should plan for its children is then

$$C_{t+1}{}^{is} = C_t^{is} \tag{4.23}$$

Let the parents' consumption propensity that satisfies (4.23) be g^{is}, which their children inherit by assumption [A3]. The parents pay $g^{is}S_t(D-1)/(D+1)$ in consumption taxes, but receive a lump sum (using (4.16) and (4.17)) of $\tilde{\phi}\tilde{g}S_t = (D-1)S_t/(D+2)$. The bequest they pass on to each of their two children is therefore

$$\begin{aligned}&[1-g^{is}-(D-1)g^{is}/(D+1)+(D-1)/(D+2)]\Gamma S_t = \\ &\qquad [(2D+1)/(D+2)-2Dg^{is}/(D+1)]\Gamma S_t\end{aligned} \tag{4.24}$$

This is averaged with their child's mate's bequest, which is $\Gamma S_t/(D+2)$ (from (18)) because the parents are assumed to be acting alone. Their child's actual consumption propensity when mated will be $(g^{is}+\tilde{g})/2$, the average of his and his mate's inherited propensities, so from (4.23) and (4.24) we get an equation for g^{is}:

$$\begin{aligned}&C_{t+1}{}^{is} = C_t^{is} = g^{is}S_t = \\ &\quad [g^{is}+(D+1)/(D+2)]\Gamma S_t[(2D+1)/(D+2)-2Dg^{is}/(D+1)+1/(D+2)]/4\end{aligned}$$

$$\Rightarrow [D/(D+1)](g^{is})^2 - [1/(D+2)]g^{is} - (D+1)^2/(D+2)^2 = -(2/\Gamma)g^{is} \tag{4.25}$$

To achieve **external sustainedness**, the per capita consumption level that the particular family should plan for its children is

$$C_{t+1}{}^{es} = \tilde{C}_t = (D+1)S_t/(D+2).$$

Therefore (4.25) is replaced by

$$[D/(D+1)](g^{es})^2 - [1/(D+2)]g^{es} - (D+1)^2/(D+2)^2 = \\ -(2/\Gamma)(D+1)/(D+2) \tag{4.26}$$

Given that g^{es}, g^{ss}, g^{is} and $\tilde{g}$ are respectively defined by (4.26), (4.22), (4.25) and (4.17), Lemma A4.5 in the Appendix shows that if

$$\Gamma > 8D(D+2)/(2D+1)^2 \tag{4.27}$$

(which is needed to make external sustainedness possible), then

$$\tilde{g} > g^{is} > g^{ss} > g^{es}. \tag{4.28}$$

One immediate corollary of these inequalities is that neither internal nor external sustainedness can be attained exactly if everyone aims for them as individuals: if everyone chooses g^{is}, the economy falls short of sustainedness, while if everyone chooses g^{es}, the economy goes beyond it ($U_{t+1} > U_t$).

Given the above analysis for a family acting alone, what will happen if *all* families are simultaneously contemplating individual sustainedness action? We give here a very preliminary analysis of the multi-person, continuous strategy, one-shot game that any one family (say family *f*) would then play by considering just its own payoffs. It assumes that all other families do the same thing as each other, but not necessarily the same as it, family *f*, does. This short-cuts a formal analysis of all the possible strategic interactions of one family with N others, and of then letting N approach infinity. We also take the short cut of assuming for the moment that all families make discrete choices of g and $\bar{g}$ among only four values of g, namely the zero-B social optimum $\tilde{g}$, the internal sustainedness value g^{is}, the social sustainedness value g^{ss}, and the external sustainedness value g^{es}.

This game produces quite complex results, as illustrated for four examples of parameter value combinations in **Table 4.1** which was computed by spreadsheet using base 10 logarithms. Each cell in the four, 4x4 matrices in the table gives the total welfare W_t (including the sustainedness value B^{is} or B^{es}, as appropriate, and with $(1+1/D)\log\Gamma$ added throughout to make all figures positive) for one of the parents in family f that results from f's choice of a consumption propensity g, depending on the propensity $\bar{g}$ chosen by the rest of society. We have to consider all *ex ante* possibilities that family f chooses a different g from the rest of society, even though *ex post* it will end up choosing the same g, so that only the matrix cells on the main diagonal can possibly be equilibria for society. In the table we abbreviate $\tilde{g}$, g^{is}, etc to $\sim$, *is*, etc; and hereafter we talk of 'playing $(\sim, is)$' to mean family f choosing a consumption propensity $\tilde{g}$ while all other families choose g^{is}. Similarly, $PV(ss,ss) := U_t^{ss} + U_{t+1}^{ss}/D$, etcetera. In the first matrix outcomes which are internally or externally sustained are marked (i) or (e) respectively; the same categorisation would apply to the other three matrices, since it is only the value of sustainedness that varies between them.

All four examples in Table 4.1 have the same pair of parameter values, $\Gamma=2.5$ and $D=6$. For this pair, the zero-B social optimum (the $\sim,\sim$ equilibrium) is unsustained by quite a margin ($D+2-\Gamma = 5.5 > 0$), but external sustainedness can be attained by individual action because Γ satisfies (4.27). What varies between examples is the precise value B^{is} or B^{es} that family f places on internal or external sustainedness. In all examples the socially sustained (ss,ss) cell is the global social optimum, being the highest cell on the principal diagonal, so a government SD policy which takes us from $(\sim,\sim)$ to (ss,ss) is thus apparently worthwhile. In each

Table 4.1 Examples of total welfare payoffs to individual family's choice of consumption propensity

(Suffix of) consumption propensity g that family chooses		(Suffix of) propensity $\bar{g}$ that family f thinks all other families will choose: $\sim$	is	ss	es
Example 1: Parameter values are	$\sim$	.312	**<u>.319</u>**	**<u>.320</u>**	**<u>.321</u>**
$\Gamma=2.5$, $D=6$	is	**<u>.338</u>** (*i*)	.225	.224	.214 (*e*)
[$\Rightarrow PV(\sim,\sim)-PV(ss,ss)=.107$]	ss	.321 (*i*)	.314 (*i*)	.313 (*i*,*e*)	.195 (*e*)
$B^{is}=.108$ $B^{es}=0$	es	.118 (*i*,*e*)	.106 (*i*,*e*)	.104 (*i*,*e*)	.087 (*i*,*e*)
Example 2: Parameter values are	$\sim$	.312	.319	.320	**<u>.321</u>**
$\Gamma=2.5$, $D=6$	is	**<u>.348</u>**	.225	.224	.214
$B^{is}=.118$	ss	.331	**<u>.324</u>**	**<u>.323</u>**	.195
$B^{es}=0$	es	.127	.116	.114	.097
Example 3: Parameter values are	$\sim$	**<u>.312</u>**	**<u>.319</u>**	.320	.321
$\Gamma=2.5$, $D=6$	is	.230	.225	.224	**<u>.332</u>**
$B^{is}=0$	ss	.213	.207	**<u>.323</u>**	.313
$B^{es}=.118$	es	.127	.116	.114	.097
Example 4: Parameter values are	$\sim$	.312	.319	.320	.321
$\Gamma=2.5$, $D=6$	is	.230	.225	.224	**<u>.536</u>**
$B^{is}=0$	ss	.213	.207	**<u>.527</u>**	.517
$B^{es}=0.322$	es	**<u>.332</u>**	**<u>.320</u>**	.318	.301

matrix the highest cell in each column has been underlined, to show what will be family f's best response to the rest of society's choice. Since the game is symmetric, any underlined cell on the principal diagonal represents a Nash equilibrium of the game, where each family is making its best response to what all other families choose.

In *Example 1*, where the value of internal sustainedness (.108) is only just enough to make (*ss*,*ss*) worth more than ($\sim$,$\sim$), there is no Nash equilibrium visible in the table, so further investigation would be necessary (e.g. looking at choices of g intermediate between the four discrete values, or at probabilistic combinations of the four) to determine the outcome of the game. However, given that a family's best response is always at least g^{is} ($> g^{ss}$ by (4.28)), it seems highly unlikely that the global social optimum (*ss*,*ss*) will be the equilibrium selected.

By contrast, in *Example 2*, the value of internal sustainedness (.118) is high enough to make (*ss*,*ss*) a Nash equilibrium. Moreover, it should be attainable starting from a 'zero action focal point' of ($\sim$,$\sim$), since for $\bar{g} > g^{ss}$ each family's best response is consume less than other people are consuming, so driving society's consumption propensity down to g^{ss}. However, in *Example 3* where it is external sustainedness which is valued at .118, ($\sim$,$\sim$) is now also a Nash equilibrium. It also seems the more likely equilibrium to be selected, for even if everyone else consumed a little less than $\tilde{g}$, it would still be in an individual family's interest to stick at $\tilde{g}$. Only if the value of sustainedness is much higher, such as .322 in *Example 4*, does (*ss*,*ss*) become the sole Nash equilibrium, and one that would be attainable by individual action, starting from a focal point of ($\sim$,$\sim$).

One cannot draw any hard and fast conclusions from so few calculations of a solution based on a particular functional form, but this preliminary analysis suggests that:

(a) There will be a range of sustainedness values in Case IIIA for which individual action to reach sustainedness will not happen (assuming rational individuals), but a collective SD policy will be both feasible and desirable, as in Example 1;

(b) The range may be finite. If so, placing an *infinite* collective value on sustainedness is inconsistent with individual preferences, since the finite range means that if individuals had unbounded concern for sustainedness, they would achieve it through individual action.

(c) The range will be wider for external than for internal sustainedness. That is, the sustainedness value that would motivate individual parental action and render a government SD policy unnecessary has to be much greater if parents seek external rather than internal sustainedness. Paradoxically, if parents aimed for less (merely to make their children as well off as they are, rather than as the whole world is), they would achieve more, since the former aim ends up achieving the latter aim as well in equilibrium!

(d) In any case, as shown in Lemma A4.5 in the Appendix, the resource growth rate may be too low for external sustainedness to be individually feasible. If this is the type of sustainedness people are aiming for, collective policy will then be the only way to achieve it, and there potentially *is* an infinite value of sustainedness which can be invoked to justify such policy, in contrast to point (b).

4.4 THE EQUIVALENT MODEL IN AN ASEXUAL WORLD

Economic growth theory almost always ignores mating-bequest externalities, and assumes asexual agents who either have infinite lives or reproduce by parthenogenesis. We therefore need to check what results of the sexual model carry across to its asexual, reduced form. So let us now modify the model in Section 4.3 by ignoring sex: each agent in generation t simply reproduces itself. Everything else – a homogeneous population of constant size, and exogenous utility discount and resource growth factors – is unchanged. For reasons that will become clear, we label the discount factor used here as D_a, to distinguish it from the factor used in Section 4.3. As before, agent t chooses a consumption propensity g, giving consumption

$$C_t = gS_t. \tag{4.29}$$

Suppose – even though there is apparently no welfare justification for it – that the government taxes consumption at rate ϕ, so the tax payment is $\phi g S_t$, and also gives a lump sum payment of $\phi \bar{g} S_t$, where $\bar{g}$ is the population average consumption propensity. The bequest left to agent t's child is then $(1-g-\phi g+\phi\bar{g})\Gamma S_t$. As with the sexual model, an agent assumes it cannot influence its child's consumption propensity, which we label y, so:

$$C_{t+1} = y(1-g-\phi g+\phi\bar{g})\Gamma S_t. \tag{4.30}$$

Ignoring any sustainedness concerns for the moment, agent t's *ex ante* perception of its welfare is thus

$$W_t(g) = \log(gS_t) + (1/D_a)\log[y(1-g-\phi g+\phi\bar{g})\Gamma S_t]$$

which is maximised by choosing g s.t.

$$\partial W_t/\partial g = 1/g - (1+\phi)/[D_a(1-g-\phi g+\phi\bar{g})] = 0.$$

Since all agents are identical *ex post*, we may set $\bar{g} = y = g$,

$$\Rightarrow g = \hat{g} := D_a/(D_a+1+\phi)$$

is the privately optimal consumption propensity.[12] From (4.29) and (4.30) the corresponding levels of consumption are

$$\hat{C}_t = D_a S_t/(D_a+1+\phi); \quad \hat{C}_{t+1} = (1+\phi)D_a\Gamma S_t/(D_a+1+\phi)^2 \tag{4.31}$$

From the government's perspective, there are no externalities in an asexual world, so the optimal tax rate is zero: $\phi = \tilde{\phi} := 0$, and the zero-B free market equilibrium and zero-B social optimum are identical:

$$g^\mu = \tilde{g} = D_a/(D_a+1) \tag{4.32}$$

$$C_t^\mu = \tilde{C}_t = D_a S_t/(D_a+1); \quad C_{t+1}{}^\mu = \tilde{C}_{t+1} = D_a\Gamma S_t/(D_a+1)^2. \tag{4.33}$$

However, suppose each agent now has a discontinuous concern for sustainedness, so that its welfare function is

$$\begin{aligned} W_t(g) &= \log(gS_t) + (1/D_a)\log[g(1-g-\phi g+\phi\bar{g})\Gamma S_t] \\ &\quad + BJ\{g(1-g-\phi g+\phi\bar{g})\Gamma, g\} \end{aligned} \tag{4.34}$$

From (4.33), this makes no difference if $\Gamma > D_a+1$, since the economy is then already sustained. But if $\Gamma < D_a+1$, from (4.31) sustainedness can be achieved by the government applying a consumption tax at rate given by

$$(1+\phi)\Gamma/(D_a+1+\phi) = 1$$

$$\Rightarrow \phi = \phi^{ss} := (D_a+1-\Gamma)/(\Gamma-1) \qquad (4.35)^{13}$$

12. A useful variant is $1-\hat{g} = (1+\phi)/(D_a+1+\phi)$.

13. A useful variant is $D_a/(D_a+1+\phi^{ss}) = (\Gamma-1)/\Gamma$.

The sustained solution is as in the sexual model:

$$\Rightarrow g = g^{ss} = (\Gamma-1)/\Gamma$$

$$C_t^{ss} = \hat{C}_{t+1} = D_a S_t/(D_a+1+\phi^{ss}) = (\Gamma-1)S_t/\Gamma$$

But even if the value B of sustainedness is high enough, namely

$$B > (\tilde{U}_t+\tilde{U}_{t+1}/D) - (U_t^{ss}+U_{t+1}{}^{ss}/D)$$

one cannot apparently justify the sustainedness tax (4.35) as an act of public policy, since if $\phi=0$ in (4.34), each agent will apparently be free to choose a propensity g^{ss} to maximise its individual welfare.

However, *the asexual model is only a reduced-form representation of the real sexual world we live in.* Mating-bequest externalities are unavoidable in practice, and perhaps cannot be internalised by individual initiatives, as shown in Section 4.3.3. Provided $B > (\tilde{U}_t+\tilde{U}_{t+1}/D) - (U_t^{ss}+U_{t+1}{}^{ss}/D)$, so that people care enough about sustainedness, there is a reason, albeit a hidden one, to analyse a SD policy using an asexual model of the world. This is about the most important message of this chapter, and it explains for example why sustainedness was a policy goal worth talking about in Chapter 3, where there were no apparent externalities caused by resource depletion.

4.5 COMPARING RESULTS FROM THE SEXUAL AND ASEXUAL MODELS

Table 4.2 now compares key results from Sections 4.3 and 4.4. The alternative sets of results for the asexual model that are given in the double-outlined box will be explained soon. In all other parts of the table, note that the sexual results can be derived from the asexual results by substituting

$$D_a = 2D \qquad (4.36)$$

Table 4.2 Comparing results from sexual and asexual models

<table>
<tr><th>Variable</th><th>Sexual model of Section 4.3</th><th colspan="2">Asexual model of Section 4.4</th></tr>
<tr><td>$\hat{g}$ = consumption propensity given tax rate ϕ</td><td>$2D/(2D+1+\phi)$</td><td colspan="2">$D_a/(D_a+1+\phi)$</td></tr>
<tr><td>$\hat{C}_{t+1}/\hat{C}_t$ = intergen'l consumption ratio given ϕ</td><td>$(1+\phi)\Gamma/(2D+1+\phi)$</td><td colspan="2">$(1+\phi)\Gamma/(D_a+1+\phi)$</td></tr>
<tr><td>g^μ = free market consumption propensity</td><td>$2D/(2D+1)$</td><td colspan="2">$D_a/(D_a+1)$</td></tr>
<tr><td>C_{t+1}^μ/C_t^μ = intergen'l free mkt consumption ratio</td><td>$\Gamma/(2D+1)$</td><td colspan="2">$\Gamma/(D_a+1)$</td></tr>
<tr><td></td><td></td><td>From asexual model</td><td>By analogy with sexual model</td></tr>
<tr><td>$\tilde{\phi}$ = zero-B socially optimal consumption tax rate</td><td>$D/(D+1)$</td><td>0</td><td>$D_a/(D_a+2)$</td></tr>
<tr><td>$\tilde{g}$ = zero-B socially optimal consumption propensity</td><td>$(D+1)/(D+2)$</td><td>$D_a/(D_a+1)$</td><td>$(D_a+2)/(D_a+4)$</td></tr>
<tr><td>$\tilde{C}_{t+1}/\tilde{C}_t$ = zero-B socially optimal consumption ratio</td><td>$\Gamma/(D+2)$</td><td>$\Gamma/(D_a+1)$</td><td>$2\Gamma/(D_a+4)$</td></tr>
<tr><td>ϕ^{ss} = consumption tax rate to achieve sustainedness[1]</td><td>$(2D+1-\Gamma)/(\Gamma-1)$</td><td colspan="2">$(D_a+1-\Gamma)/(\Gamma-1)$</td></tr>
<tr><td>g^{ss} = sustainable consumption propensity</td><td>$(\Gamma-1)/\Gamma$</td><td colspan="2">$(\Gamma-1)/\Gamma$</td></tr>
<tr><td>C_{t+1}^{ss}/C_t^{ss} = sustainable consumption ratio</td><td>1</td><td colspan="2">1</td></tr>
</table>

1 SD policies are assumed to apply only if there is a sustainedness problem in the zero-B optimal solution, i.e. if $\Gamma < D + 2$ or $\Gamma < D_a/2 + 2$.

So *as long as the intergenerational utility discount factor used in the asexual model is* **twice** *that in the sexual model, the asexual model's results for the tax, free market and sustained equilibria also apply to a sexual world.* This appears not to have been noted before in the economic literature, although it will not be surprising to geneticists, who typically assume that altruism is weighted by the proportion of genes shared in common. If any sexual models were ever to get used for policy purposes, they therefore could not use utility discount rates estimated from asexual models (e.g. from simple survival probabilities, as in Kula 1984).

In the double-lined box in Table 4.2, the asexual model's results do not apply in the sexual model, even after discount factor adjustments. There is no formal bequest externality in the asexual model, and so no need for a corrective tax. However, since the asexual model is trying to mimic reality, we must replace these results with results for $\widetilde{\phi}$, $\widetilde{g}$ and $\widetilde{C}_{t+1}/\widetilde{C}_t$ derived by analogy from the sexual model, using (4.36); these are listed in the rightmost column. So although the sustainedness tax $\phi^{ss} = (D_a + 1 - \Gamma)/(\Gamma - 1)$ appears unnecessary in an asexual model because there is no formal mating-bequest externality, we can still regard it as a justified policy intervention if we believe that parents are sufficiently concerned about sustainedness, but are unable or unwilling to achieve it individually in the real, sexual world.

4.6 CONCLUSIONS

In any species, parents are concerned for their children's wellbeing, otherwise reproduction and life would cease. In a sexual species, children mate outside the family, ultimately for good genetic reasons. If such mating is at random, being guided by 'blind love' rather than arrangements between families already known to each other, parents ignore the fact that increasing their bequests to their children makes the parents of their children's mates happier. It is then well-known that this causes parental bequests of resources to be generally less than is socially desirable, and policies such as consumption taxes or bequest subsidies can increase social welfare.

This chapter has extended this insight to include parental concern for family sustainedness. For the sake of simplicity this was treated as a discontinuous jump in welfare which parents enjoy, if they know their children will be at least as well off as they themselves or as the current generation are. This concern was added to the concave, continuous welfare which it is normally assumed that parents derive from their children's hedonistic utility, and applied to a non-overlapping generations model where each generation has the same number of identical, opposite-sex couples, who each own an exogenously growing resource stock which can be either consumed now or bequeathed to the next generation. By assuming particular functional forms for resource growth and welfare, we illustrated how a public policy goal of achieving sustainedness *can* be based on the preferences of individual couples for the sustainedness (i.e. non-declining utility) of their own family line, and is not necessarily an undemocratic and therefore unimplementable objective, as Kennedy (1994) claims. We also highlighted the need for empirical and philosophical research on the structure of parental concerns for their children's utility, since little is known about such concerns in practice.

The same fiscal instrument (a consumption tax) can be used to increase parental bequests and thus fully internalise either the continuous or discontinuous concern that parents have for their children's utility. Policies for 'zero-B optimal' bequests (i.e. optimal ignoring individual's concerns for sustainedness) and for sustainability-preserving bequests therefore overlap in the same way as environmental and SD policies overlapped in Chapter 2. Zero-B optimal policies may achieve sustainedness as a side-effect, but if they do not, an explicit SD policy may then be possible, if the current generation has the power of 'infinite future commitment' and can guarantee that future governments or its offspring will have the same attitude towards sustainedness that it does now. And such a policy will be *justified* if two further conditions hold. Firstly, the discontinuity in parents' concern – the 'value of sustainedness' – must be large enough to warrant the reduction in the present value of the continuous part of utility that an SD policy will cause.

Secondly, it must be either impossible or inefficient for individuals to achieve sustainedness through their own initiative, a condition similar to that underlying Conjecture 2.1 in Chapter 2. A preliminary game-theoretic analysis of parents' choices of consumption propensity showed that this condition can be true in some cases of our model; but in other cases, individual sustainedness is possible and efficient, so sustainedness cannot have infinite value and therefore be an over-riding *policy* concern, as individuals would then achieve it on their own anyway. The analysis also showed that individual sustainedness is more likely to occur if parents aim for their children to be as well off as the average in the current generation (called 'external sustainedness'), rather than just to be as well off as they (the parents) are ('internal sustainedness') – a somewhat paradoxical result, as the same overall state of the economy achieves both aims at once.

Since economic growth theory almost always ignores mating-bequest externalities by assuming asexual agents who either have infinite lives or reproduce by parthenogenesis, we checked what results of the sexual model carry across to its asexual, reduced form. Several formulae remain the same, including the tax level needed to attain sustainedness, save only that the utility discount factor used in the asexual model must be *twice* the sexual discount factor. And while an asexual model of purely private resource depletion with no environmental effects predicts that no government SD policy can be justified in terms of individual preferences, this may well be false in the more realistic, sexual model. So if we are willing to assume that individuals desire non-declining utility for their descendants, and cannot in practice achieve it efficiently (or at all) by individual increases in bequests, the comparison of these two simple models strongly suggests a cost-benefit rationale for using government policy to achieve sustainedness – even when, as in Chapter 3, it is analysed in an asexual model apparently 'free' of externalities. As long as sex and death exist, the freedom is always only apparent, and never real.

APPENDIX TO CHAPTER 4

Lemma A4.1

$\log x > 1 - 1/x$ if $x > 1$.

Proof:

For $t > 1$, $1/t > 1/t^2$

$\Rightarrow \int_1^x (1/t)dt > \int_1^x (1/t^2)dt$ for $x > 1$

$\Rightarrow [\log x]_1^x = \log x > [-1/t]_1^x = 1 - 1/x$. ||

Lemma A4.2

$y\log(1+\epsilon/y)$ is an increasing function of y, for $\epsilon > 0$ and $y > 0$.

Proof:

$$\begin{aligned}(d/dy)[y\log(1+\epsilon/y)] &= \log(1+\epsilon/y) + y(-\epsilon/y^2)/(1+\epsilon/y)\\ &= \log(1+\epsilon/y) - (1+\epsilon/y-1)/(1+\epsilon/y)\\ &= \log(1+\epsilon/y) - [1-1/(1+\epsilon/y)]\\ &> 0 \text{ by putting } x = 1+\epsilon/y \text{ in Lemma A4.1.}\end{aligned}$$ ||

Lemma A4.3

$$(1+1/D)\log[(D+1)S_t/(D+2)] + (1/D)\log[\Gamma/(D+2)] >$$
$$(1+1/D)\log[2DS_t/(2D+1)] + (1/D)\log[\Gamma/(2D+1)]$$

Proof:

By putting $y = D+x > 0$ and $\epsilon = D-1 > 0$ in Lemma A4.2, $(D+z)\log[1+D/(D+z)]$ is an increasing function of z. Hence

$(D+1)\log[1+(D-1)/(D+1)] > D\log[1+(D-1)/D]$

$\Rightarrow [(D+2)/D]\log[(2D+1)/(D+2)] > [(D+1)/D]\log[2D/(D+1)]$

$$\Rightarrow (1+1/D)\log[(2D+1)(D+1)/2D(D+2)] - (1/D)\log[(D+2)/(2D+1)] > 0$$

$$\Rightarrow (1+1/D)\log[(D+1)S_t/(D+2)] + (1/D)\log[\Gamma/(D+2)] > (1+1/D)\log[2DS_t/(2D+1)] + (1/D)\log[\Gamma/(2D+1)]$$

which from (4.15) and (4.19) means $\widetilde{W}_t > W_t^{\mu}$. ||

Lemma A4.4

$y > \log(1+y)$ if $y > 0$.

Proof:

For $t > 0$, $1 > 1/(1+t)$

$\Rightarrow \int_0^y dt > \int_0^y [1/(1+t)]dt$ for $y > 0$

$\Rightarrow [1]_0^y = y > [\log(1+t)]_0^y = \log(1+y)$. ||

Lemma A4.5

To show that if the following hold:

$$[D/(D+1)](g^{es})^2 - [1/(D+2)]g^{es} - (D+1)^2/(D+2)^2 = -(2/\Gamma)(D+1)/(D+2) \quad (4.26)$$

$$g^{ss} = (\Gamma-1)/\Gamma \quad (4.22)$$

$$[D/(D+1)](g^{is})^2 - [1/(D+2)]g^{is} - (D+1)^2/(D+2)^2 = -(2/\Gamma)g^{is} \quad (4.25)$$

$$\widetilde{g} = (D+1)/(D+2) \quad (4.17)$$

$$\Gamma < D+2 \quad \text{from } (4.20)$$

$$\Gamma > 8D(D+2)/(2D+1)^2 \quad (4.27)$$

then $g^{es} < g^{ss} < g^{is} < \widetilde{g}$. *(4.28)*

Proof:

One can readily check that the quadratic

$$f(g) = [D/(D+1)]g^2 - [1/(D+2)]g - (D+1)^2/(D+2)^2 = 0$$

has roots

$$g_1 = -(D+1)/(D+2) \text{ and } g_2 = (D+1)^2/[D(D+2)]$$

so that the graph of $f(g)$ looks like **Figure 4.3**. Between $g_1 < 0$ and $g_2 > 0$, $f(g) < 0$. The minimum point M has coordinates $\{(D+1)/[2D(D+2)], -[(D+1)(2D+1)^2/4D(D+2)^2]\}$. Hence there is at least one real positive root for g^{es} (i.e. external sustainability is feasible) only if

$$-(2/\Gamma)(D+1)/(D+2) > -[(D+1)(2D+1)^2/4D(D+2)^2]$$

from which condition (4.27) follows immediately. Assume now that (4.27) does hold, which means that

$$-2/\Gamma > -(2D+1)^2/[4D(D+2)] > -(2D+1)^2/2(D+2).$$

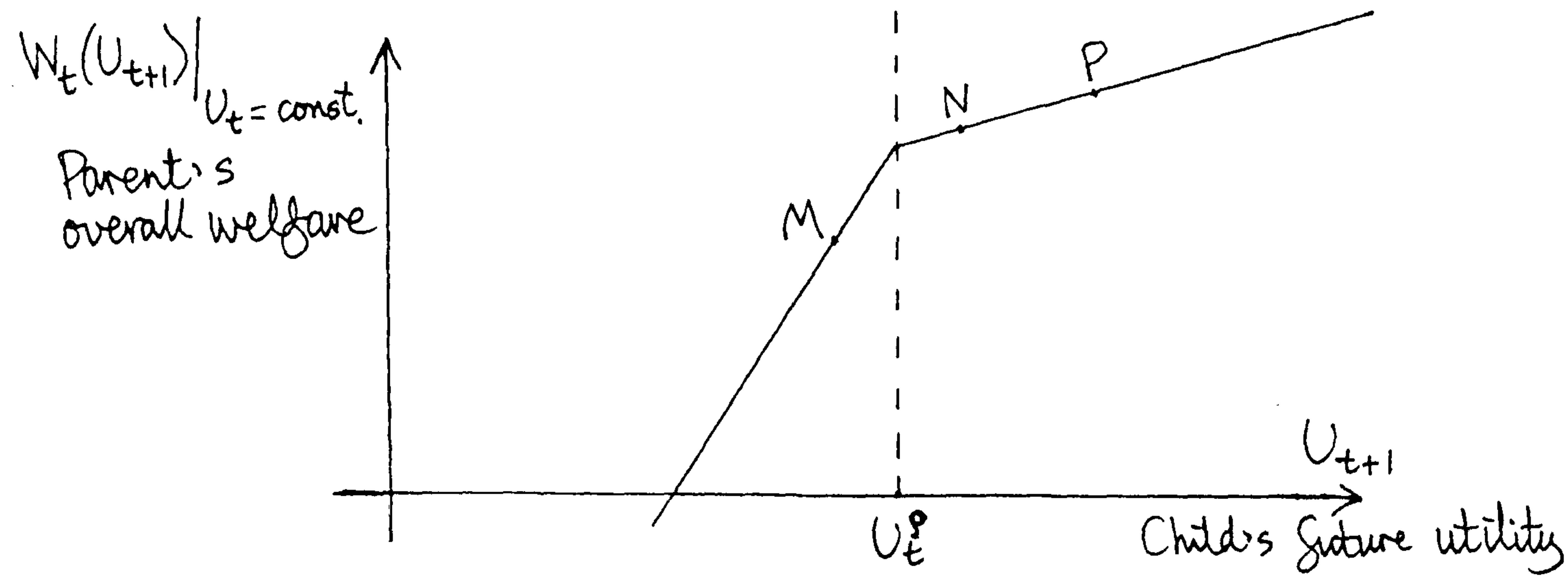
Since the last expression is the slope of OM, the line $-2g/\Gamma$ must cut the graph of $f(g)$ somewhere in the segment MQ, as shown. So g^{is} must lie in the segment PQ, where $f(g)$ is an increasing function of g; and of course the higher real root for g^{es} must also be in PQ. To prove the Lemma it will therefore suffice to show that, as shown on Figure 4.3:

(i)	$f(\tilde{g}) > -(2/\Gamma)\tilde{g}$	$(\Rightarrow g^{is} < \tilde{g})$
(ii)	$f(g^{ss}) < -(2/\Gamma)\tilde{g}$	$(\Rightarrow g^{is} > g^{ss})$
(iii)	$f(g^{ss}) > -(2/\Gamma)(D+1)/(D+2)$	$(\Rightarrow g^{ss} > g^{es})$

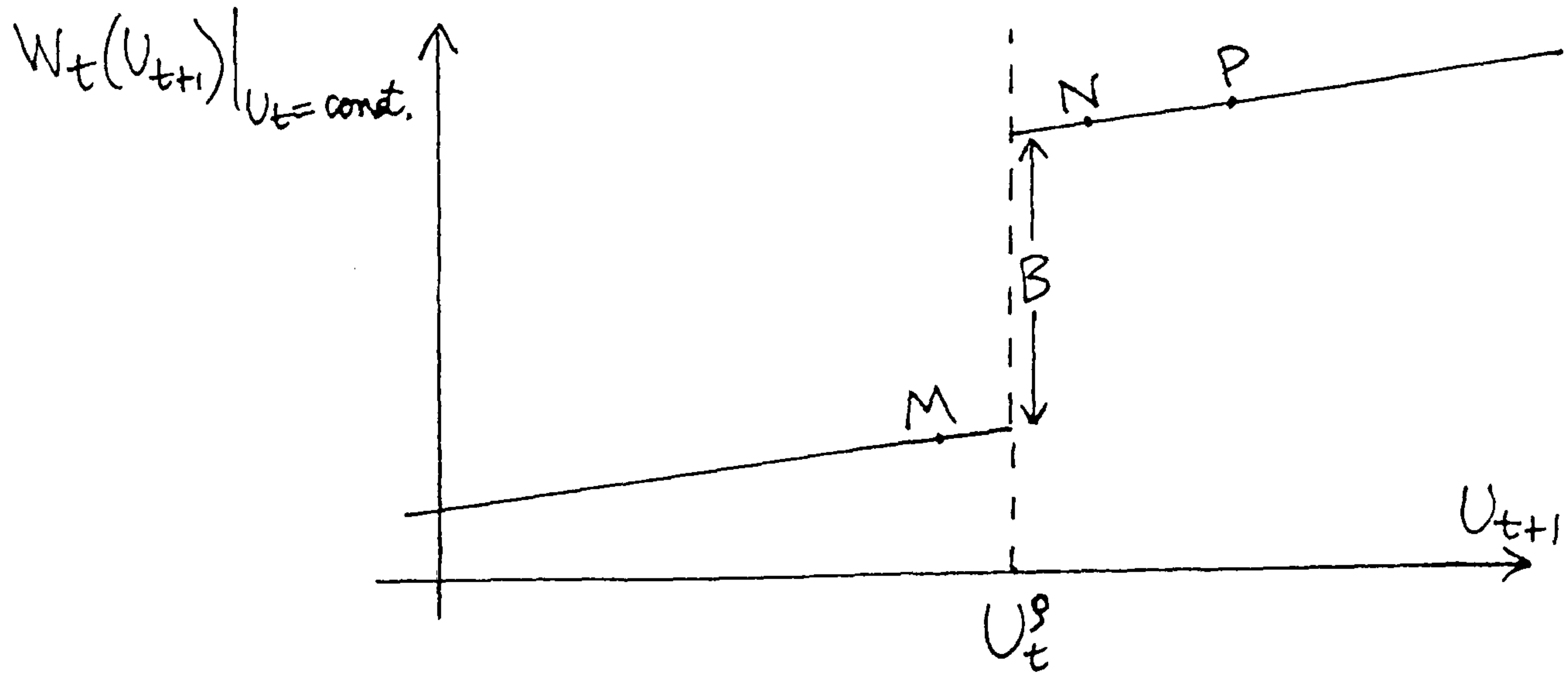
Results (i)-(iii) follow from (4.22), (4.17), (4.20) and (4.27) by routine but tedious algebra. ||

Figure 4.1 Individual parental concern for children

(a) Kinked line



(b) Stepped line



(c) Smoothed step

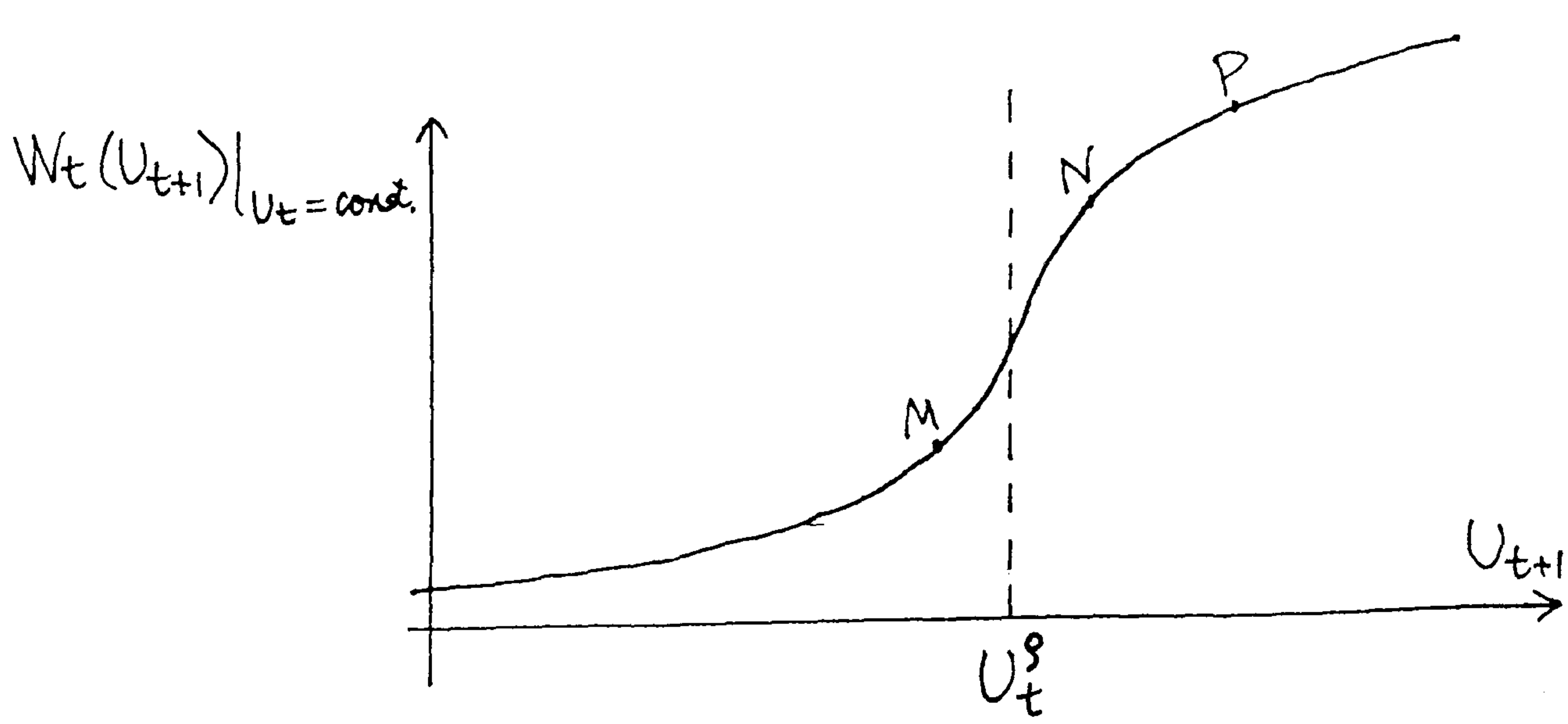
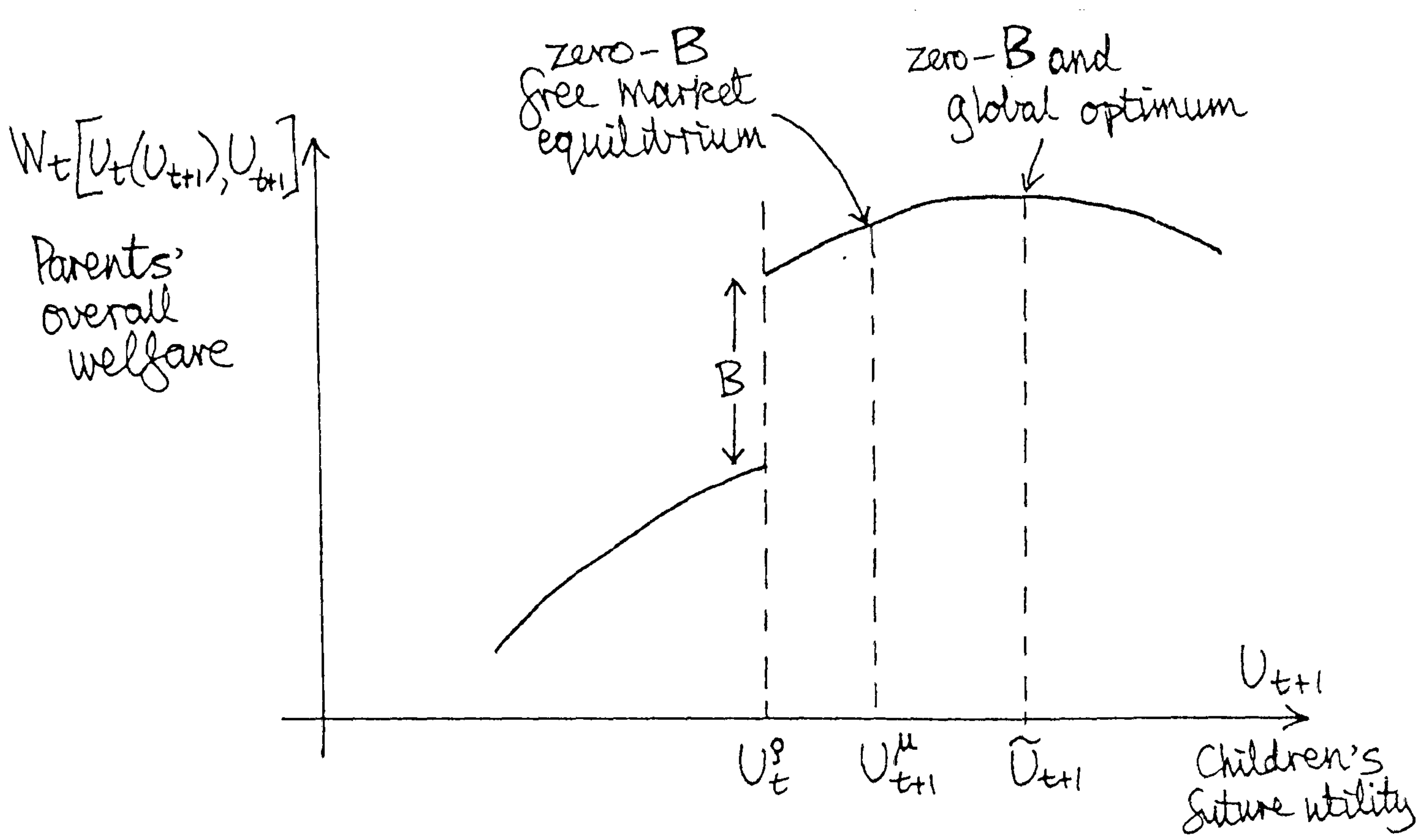
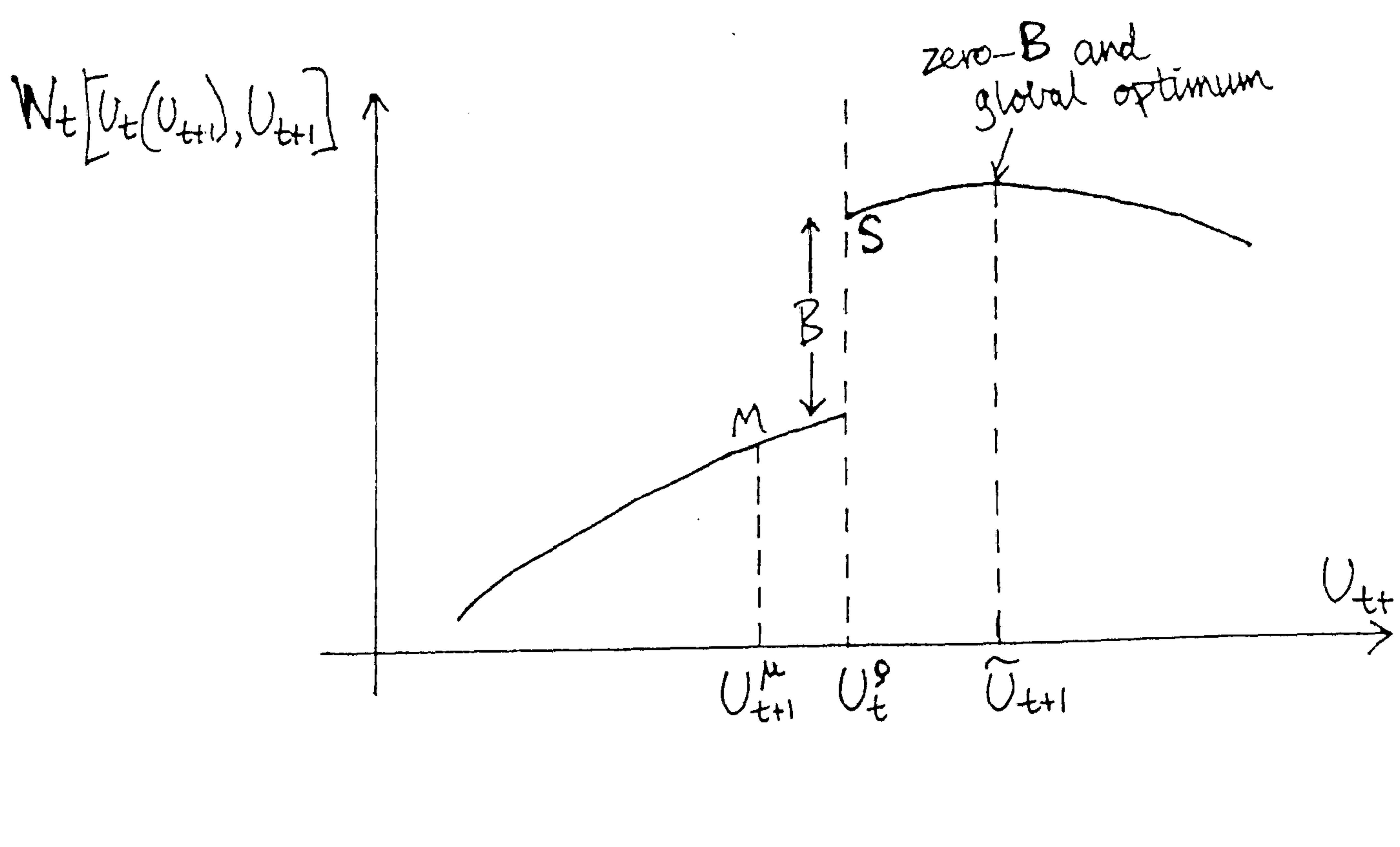


Figure 4.2 Collective parental welfare, including sustainedness concern, versus child's utility

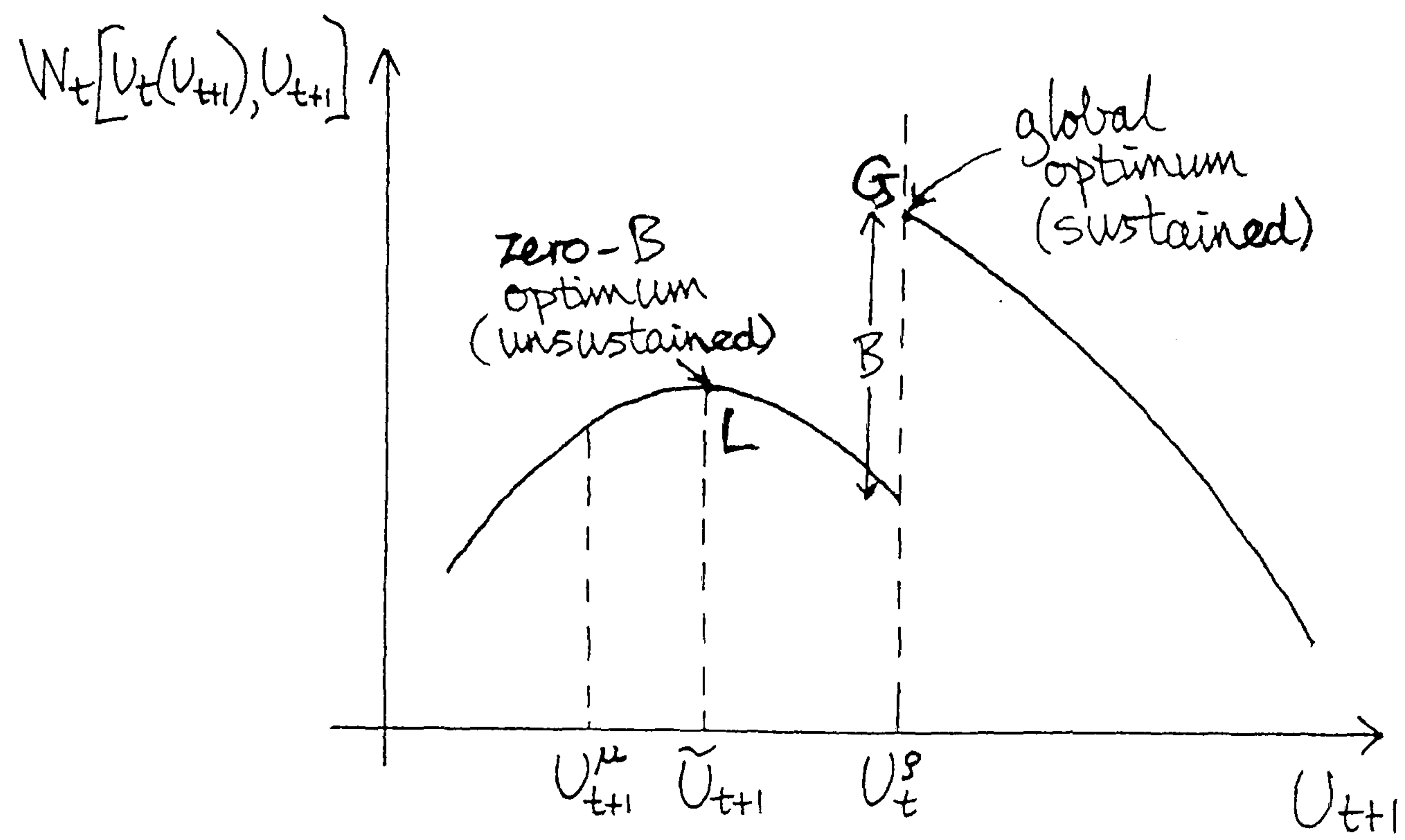
(a) Free market outcome already sustained



(b) Local social optimum already sustained



(c) Justified sustainedness policy



(d) Unjustified sustainedness policy

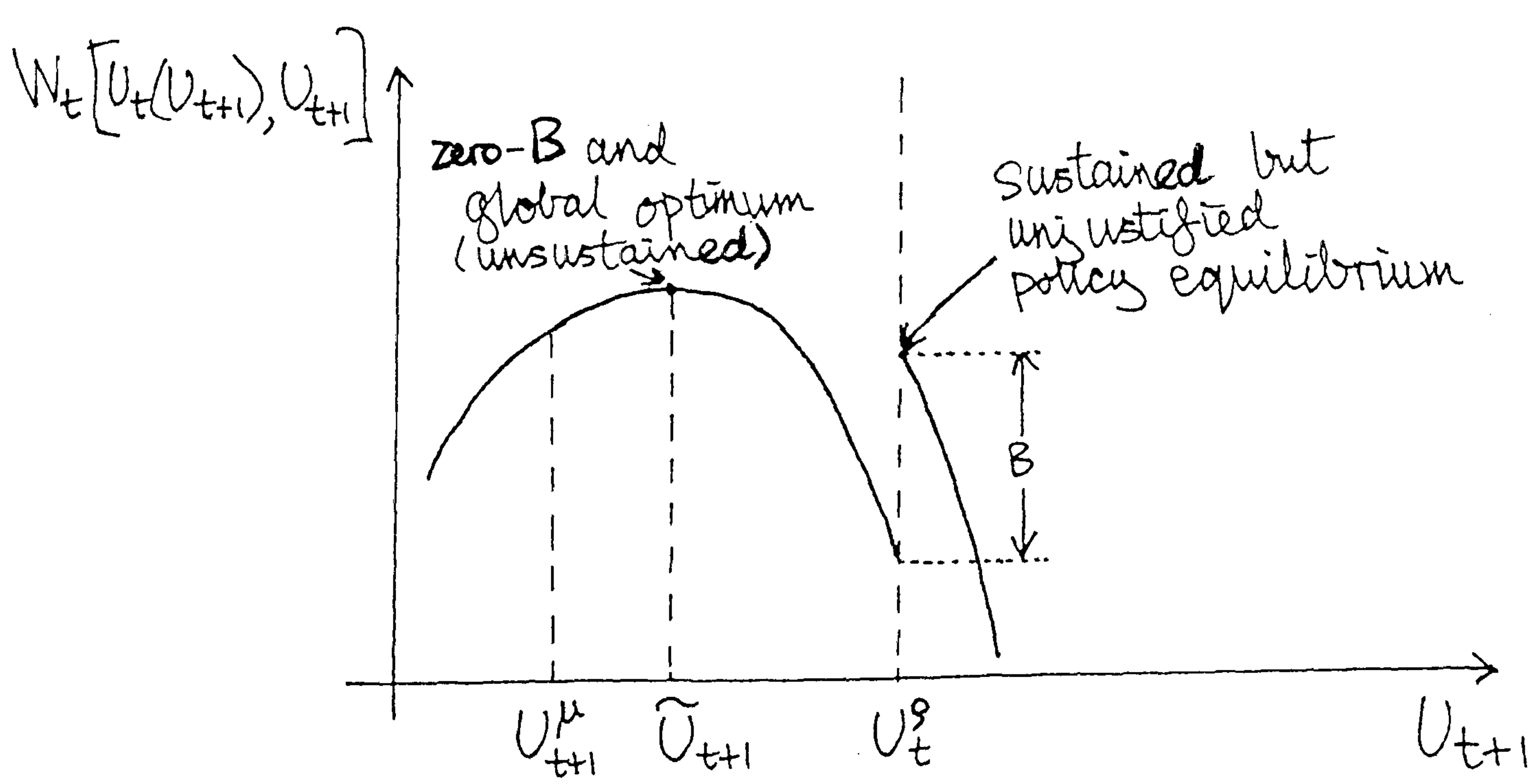
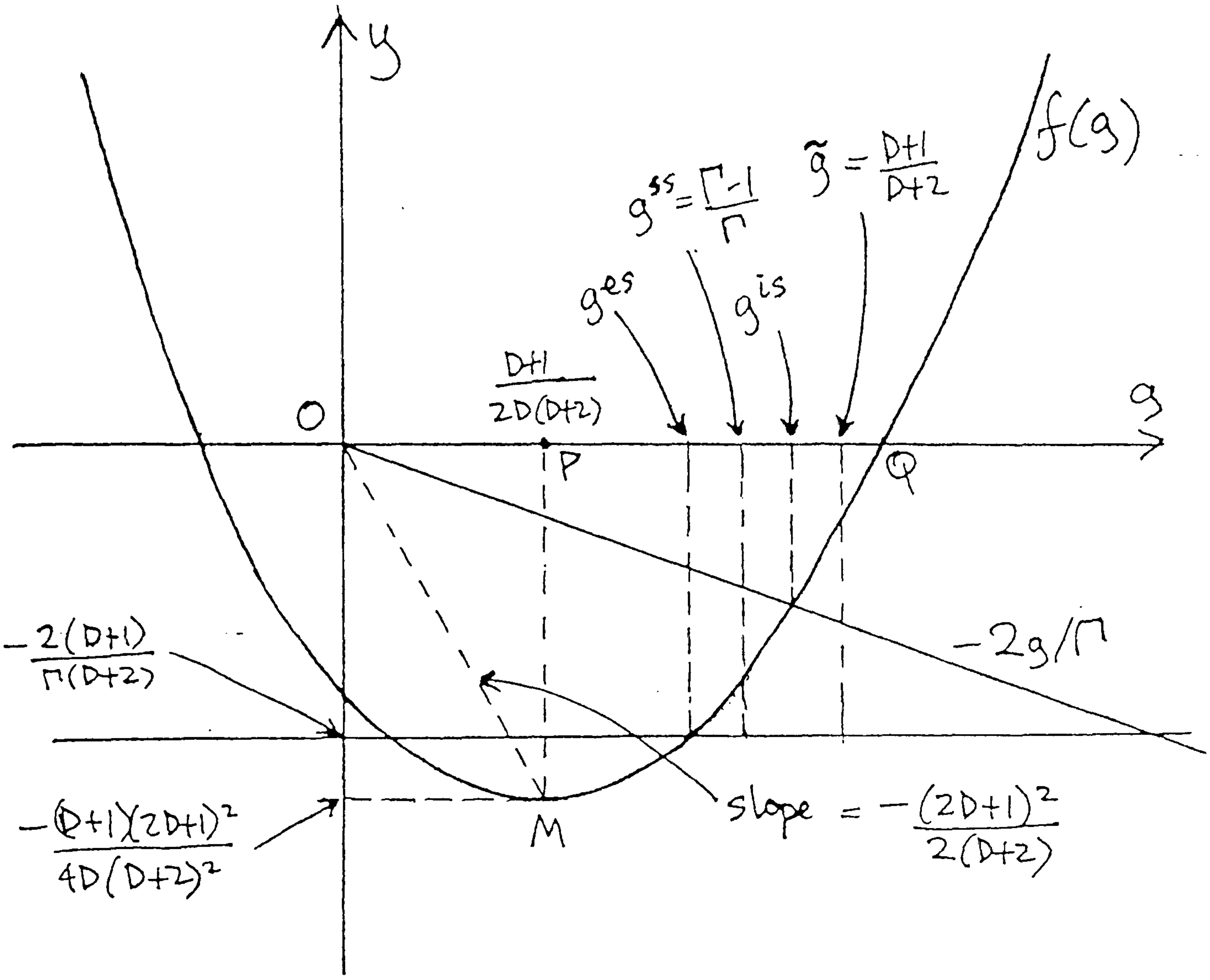


Figure 4.3 Proof of Lemma A4.5



CHAPTER 5

CONCLUSIONS TO PART I

Each of Chapters 2-4 has its own detailed concluding section. My aim here is to highlight just the more important conclusions of Part I, and their wider implications.

The main motive behind Part I was the conviction that sustainable development (SD), which numerous governments in both rich and poor countries round the world now claim to be pursuing as a goal of economic policy, can be in conflict with the conventional concept of (PV-)optimal development. Formal economic study of SD and allied concepts therefore promised to be interesting and useful. The formalism chosen was to define SD (or equivalently sustainedness) as non-declining instantaneous utility (NDU) of a representative person in a deterministic, infinite-horizon economy where natural resources are non-renewable but substitutable for by human-made capital.

The particular assumption that resources are non-renewable but substitutable made the analysis more relevant to developed countries than to developing countries. While severe problems with depletion of renewable resources in the latter have undoubtedly been the major driving force for recent interest in SD, if SD is to be a widely useful notion it should also be applicable to developed countries as well. My research should therefore be considered complementary to work aimed at developing countries.

At least five separate topics about SD have been covered. These relate closely, though not exactly, to the questions Q1-Q7 originally posed in Chapter 1, as noted in brackets:

1) How appropriate is NDU as a criterion of SD, and how does it relate to other possible criteria? (Q1, Q7)
2) How does an NDU path compare to a socially PV-optimal or free market path? (Q2-4)
3) What instruments of policy intervention can achieve NDU? (Q3-4)
4) Can an NDU policy be justified in terms of individual preferences? (Q5)
5) Does non-declining aggregate welfare guarantee NDU? (Q6)

I will address each of these in turn. It should be understood throughout that further research into the effects of different assumptions such as uncertainty, human mortality, resource renewability and non-substitutability would be desirable in most cases.

5.1 NDU VERSUS OTHER FORMAL CRITERIA OF SUSTAINABLE DEVELOPMENT

A formal mathematical criterion is often a compromise between the original, pure concept intended, and what can actually be used analytically to derive new results that are not already obvious from that concept. Defining SD as NDU is no exception to this. What has this thesis has revealed about NDU both *ex ante* in terms of how fitting its definition is to the idea of SD, and *ex post* in terms of how useful it is?

As the discussions in Chapter 1 and Section 2 of Chapter 2 recognised, NDU is clearly an oversimplification of what people and politicians really mean by 'sustainability' if they stop to think about it. One thing clear from the dozens of competing definitions of sustainability concepts catalogued in Appendix 1 of Pezzey (1989) is that they are all, at least in part, concepts of inter*generational* equity. No one is really concerned if someone chooses, in a way that affects no one else, to have declining utility at some point in his or her own lifetime. What they may well be concerned about is if the PV of a typical person's lifetime utility path declines from one generation to the next, because of the course of development that the economy is following. By omitting all distinction between young and old people alive at the same time, NDU in a representative person model conflates the two topics of potential concern. But a brief review of recent work by Howarth and Norgaard and by Mourmouras showed that non-declining PV in overlapping generations models appears much less tractable than NDU in representative person models. In particular, it would seem to be very hard to learn much about non-declining PV paths, and the policies to attain those paths, when the decline or growth of PV is not exponential. When it is, the non-declining PV and NDU criteria give analogous results. The desire for greater tractability, and to be comparable with the many other well-known works on growth with exhaustible resources, is the main reason why the I chose the NDU route.

However, the analysis of Chapters 2-4 showed that the tractability of NDU is still quite limited, as will be reviewed below. In particular, it is hard work to say much in general about how the freedom to have rising utility, which is allowed under NDU, can be used to reach the NDU path with the highest PV, which I called the 'opsustimal' path (for PV-optimal sustained). This is in contrast to the only other criterion of intertemporal allocation that has made more than a passing appearance in the thesis, which is maximin utility, effectively *maximum constant utility*. Although no one claims this to be a criterion of SD, it is certainly the best-known criterion of intergenerational equity. It is supported by a much more impressive philosophical pedigree, and moreover is a good deal more tractable than NDU, resulting in many elegant technical papers since Solow's seminal contribution in 1974. And on efficiency grounds, an opsustimal path will generally start off below the maximum constant level of utility (see for example Figure 3.5), and we know little about which path politicians would in fact prefer. But in theory it seems reasonable to suppose that they would prefer the path with higher PV, i.e. the opsustimal path, so that constant utility is less politically realistic than an NDU constraint.

It would be desirable to be even more realistic than NDU, though, by including an important part of popular concern for sustainability which NDU omits. (Recall that popular views are relevant to this thesis because it is primarily a work of political economy, not moral philosophy. It seeks to reflect intergenerational politics as it is, rather than it perhaps should be.) The omitted part is concern for the *speed* in the decline of consumption or utility, *including any declines from the initial level given by history*. If this historically given level is above the initial level of utility on the opsustimal path, it is politically unrealistic to expect an instant drop from the former to the latter to be chosen. Long-term concern for sustainability will surely be *traded off* to some extent against short-term adjustment costs. This recognised in Section 2.2, which suggested among other future research topics that the NDU criterion could perhaps be replaced by an objective of maximising, without any side constraints, a broader intergenerational social welfare function (ISWF). Such an ISWF could perhaps be the sum of discounted utility using an adjustment cost utility function $U(C,\dot{C})$, and some separate value ascribed to the asymptotic level of utility. If the elements of this broader ISWF were differentiable (which they would need to be if tractability is to be maintained), SD would become another element of the 'goodness' of a development path, rather an overriding determinant of its 'rightness' that must be preserved at all costs.

An NDU constraint is thus arguably further away than either a non-declining PV constraint, or an adjustment-cost-plus-terminal-value type of ISWF, from a politically realistic representation of SD; and further away from tractability than constant utility. But the former two alternatives are probably less tractable approaches to SD, and constant utility is less realistic, than NDU. *NDU therefore emerges from this thesis as a instructive compromise between realism and tractability in the search for a formal criterion of SD.* Whether a better compromise can be found remains for further research, probably starting with an adjustment-cost-plus-terminal-value type of ISWF approach to SD.

5.2 COMPARING AN SD PATH TO THE SOCIALLY PV-OPTIMAL OR FREE MARKET PATH

This resolves itself into two distinct issues. When does the socially PV-optimal or free market path have a decline in utility at some time? If so, what is the opsustimal (PV-optimal sustained) path, and how does it differ from the socially PV-optimal or free market path? Both these issues were examined in Chapter 2, where non-renewable resources had amenity value, and in Chapter 3, where they did not. In both cases the conditions for either the socially PV-optimal or free market path to be sustained were no great surprise. Either there has to be a source of endless, sufficiently rapid, exponential technical progress or resource growth in the economy, or capital has to be very substitutable for resource inputs. Essentially, this is because the reward for saving for the future becomes insufficient to outweigh exponential discounting of the future. This much could more or less be concluded from a critical review of the 1974 *Review of Economic Studies Symposium* (as for example provided by Toman, Pezzey and Krautkraemer 1995), although the distinction (caused by an externality in the economy) between socially PV-optimal and free market paths was absent then.

And there was some novelty in Chapter 2's demonstration that socially PV-optimal development, i.e. development where the environmental value of non-renewable resources has been internalised, may still be unsustainable, even though it will be 'more sustained' in a crude sense than the free market path which externalises this value. This finding will be news to the many policy analysts who see sustaining development and protecting the environment as virtually identical issues. More research into a quantitative measure, rather than a qualitative condition, of sustainedness would help to sharpen this result.

Another interesting insight was the conjecture (based on a numerical solution) in Chapter 3 that the opsustimal path may well initially entail higher consumption than the PV-optimal path, which is contrary to untutored intuition. The explanation is that it is PV-optimal to accumulate lots of capital early on, while resources are still plentiful and thus the marginal product of capital still high, in order to consume it in later years. An interesting topic for research would therefore be on the effect on PV-optimal and opsustimal paths of assuming non-consumable rather than consumable capital.

5.3 POLICY INSTRUMENTS TO ACHIEVE SD

The formulae in Chapters 2 and 3 for policies to achieve SD are also fairly new. The novelty is mainly because the technical literature on intergenerational equity (defined as maximum constant utility) has shown remarkably little interest in policy. So it may surprise some people to learn that even though non-renewable resource depletion and environmental degradation are key causes of unsustainable development (if it happens), *resource or environmental taxes are probably incapable of curing it.* It may also come as a surprise that the policies that do work, such as consumption and capital taxes, must end up as *subsidies* in the long run, which poses some unresolved problems for public finance.

However, such fairly clear results were available only asymptotically. During finite time, a wide range of things can happen. Active SD policies may be needed from time zero, or only after a finite time. If the latter, they must almost certainly be announced at time zero, otherwise people cannot have perfect foresight in predicting future prices, which affect actions today. In at least one case, once SD policy intervention had started, it had to continue forever, but there is no reason why this should be true generally.

5.4 JUSTIFYING SD IN TERMS OF INDIVIDUAL PREFERENCES

It would surely advance the case for treating SD as an important policy goal if it can be shown that people want SD, can achieve it through collective action (e.g. through government policy intervention), but cannot achieve it individually. Casual empiricism, allied with intuition derived from evolutionary biology and psychology, and from the academic and governmental publications on SD which I noted in Chapter 1, suggests that most people would like to see their children live as well in the future as they themselves do now. However, I did not produce any data showing that people do place a large value on SD. What I tried to do in two chapters was to advance the second part of the idea above, namely that *if* people want SD, then one can readily think of ways in which they can achieve it collectively, but not individually. The underlying intuition is clear from *Our Common Future*, the title of the book which largely inspired this thesis. If a large part of the assets which will determine future welfare are held in common, or at least not individually, then collective action may be needed to preserve sufficient assets for sustainability.

Two ways were highlighted in which assets for the future are not held individually. In Chapter 2, where individual people have minuscule influence over the quality of the future natural environment, we showed a case where collective action to conserve the environment costs an individual far less than the amount (which might be more than the individual's wealth) that individual action would cost. If people value SD in this case more than the cost of collective action, but less than the cost of individual action, then an SD policy is justified in terms of individual preferences.

In Chapter 4, individual pairs of parents are assumed to have power over only half of the material wealth that their children inherit. The other half comes attached to their children's spouses, of whom parents have no prior knowledge or control. Because of the need to model mating and bequests, we adopted an explicit generational framework with sexual rather than asexual agents in this chapter, albeit with non-overlapping generations and no distinction in sexual roles. One can then again show cases where collective action to achieve SD is cheaper than individual action, and thus potentially justified in terms of individual preferences. This particular insight is an important riposte to those (Beckerman 1994, Kennedy 1994) who regard SD as an illogical or undemocratic goal for policy in the pure intergenerational context where common environmental assets are an unimportant part of the whole. But we still lack data on how much parents are concerned for their children to be as well off as they themselves are, and any detailed analysis (whether economic, philosophical or biological) of why they should feel such concern. Both topics await further research.

5.5 NON-DECLINING AGGREGATE WEALTH AND SUSTAINABILITY

I have left until last a review of what is perhaps the most important single result of this thesis, which arose by accident. In Chapter 3, I enquired into the relationship between aggregate wealth (the combined value of all forms of human-made capital and environmental resources on a particular development path at a particular time) and sustainability (defined as the ability to achieve NDU forever, starting from the level of utility on the same path at the same time). My enquiry was confined to an economy with constant technology, and where natural resources are absent from the utility function, and present in the production function only as a flow. Pearce and his school had frequently claimed that Solow (1986) had shown non-declining aggregate wealth to be a condition for sustainability. I had intended merely to check this result in my particular models.

However, Section 3.2 showed that although non-declining wealth is necessary for sustainability in constant-technology (what were called 'Weitzman') economies, it is *not* a sufficient condition. Aggregate wealth can be rising while the current level of utility is unsustainable. Indeed, this *must* happen during some finite time interval if the economy's PV-optimal development path is single-peaked and initially sustainable. The intuition is that a general development path, including the PV-optimal one, is not subject to any kind of sustainability constraint, so there is no reason to expect relative prices on the path, which are used to calculate aggregate wealth, to tell us much about sustainability. This obviously undermines current attempts to use national accounts data to measure sustainability, although it also suggests that any negative conclusions (i.e. that a country is unsustainable if it has falling aggregate wealth measured at market prices) are strengthened.

In comparing this result with the existing literature, we then found a more serious confusion in the literature between the terms net national product (NNP) and maximum sustainable consumption. Many writers have come to assume, after reading Weitzman (1976) and/or Solow (1986), that the two terms have been shown to be equal. But *if* one defines NNP as consumption plus net investment on a *competitive* (i.e. PV-optimal) development path, *if* one defines maximum sustainable consumption as what I have termed the 'current maximin' consumption, and *if* one assumes an economy in which the PV-optimal path varies over time and is unique, then maximum sustainable consumption is always strictly less than the NNP. Research in this area therefore needs to focus on price differences between sustainable and unsustainable constant utility paths, in addition to its existing focus on the adjustments that need to be made to include environmental values in national income accounts.

PART II

CHAPTER 6

INTRODUCTION TO PART II

In a competitive industry with pollution-control costs that vary among firms, it is well known that market mechanisms such as emission charges and marketable emission permits are efficient in the short run. They equalise marginal control costs across all firms, and thus achieve a given total emission reduction at minimum cost to the industry. This was demonstrated for charges by Baumol and Oates (1971), and for permits by Montgomery (1972). But until recently, little progress was being made in introducing such minimum cost mechanisms into environmental policies. During two years spent in the mid-1980s as an Economic Adviser in the U.K. Department of the Environment, when my job was to promote market mechanisms, I came into contact with some of the arguments causing this lack of progress, arguments which are documented in Pezzey (1988).

In particular, it seemed impossible to find a form of emission charge which was both politically feasible and economically respectable. Any proposal for a pure, Pigovian emission charge – as distinct from a 'redistributive' charge at a rate so low that it would have little effect on emissions – met with strong resistance from British industry, on the grounds that 'residual' emissions (i.e. those less than current regulatory standards) were 'acceptable' and therefore should not be taxed. But including any kind of offsetting subsidy element, so that firms would still face the same marginal charge on their emissions, but would receive a per unit subsidy for emissions below a certain baseline, was criticised by economists. It was held that any subsidy element would increase firms' profits and thus encourage excessive long run entry of firms into the polluting industry (Baumol and Oates 1988, Chapter 14).

On the other hand, marketable permits seemed to stand a better chance of being both politically and economically acceptable. Evidence from the USA already suggested that political resistance to permits could be greatly reduced by giving away permits free to existing polluters ('grandfathering' them); but since permits would not be free for new polluting firms, there would be no problem of excessive entry. This lack of symmetry between 'control by price' (charges and subsidies) and 'control by quantity' (permits) struck me as strange. At a fundamental level, my instinct was that the short run symmetry between the two control systems (i.e., both achieve control at minimum cost, as noted above) should be extendable to a long run symmetry. Might the answer might lie in the nature of the *emission rights*, implying various kinds of *ownership* of the environment, which underlie the two types of control? Furthermore, might the distribution of emission rights among important political interest groups contain the key to implementing of market mechanisms as actual policies?

Chapters 7 and 8 are the result of pursuing these two questions.[1] **Chapter 7**, "The Symmetry between Controlling Pollution by Price and Controlling it by Quantity" focuses on the first question. It examines the property rights underlying various market mechanisms, and in particular studies how each mechanism treats a new firm entering a polluting industry, and an old firm leaving the industry. It shows that if the underlying emission rights are complete for both control by price and control by quantity, and if also several simplifying assumptions hold about perfect

[1] Both Chapters were written as independent essays. Chapter 7 was written in 1991, and it is printed here very much as published by the *Canadian Journal of Economics* in 1992, except for changes to section numbering. Since each Chapter has a detailed literature review, only selected references are given in this Introduction.

competition and information, then any control by price is indeed symmetrical to a control by quantity which embodies the same amount of emission rights, in that it achieves both short run *and* long run economic efficiency in resource allocation.

Chapter 8, "On the Political and Informational Economy of Distributing the Efficiency Gains from Market Mechanisms of Pollution Control", uses this economic symmetry and assumes a general market mechanism of control, mostly without specifying whether the immediate instrument of control is a price (the emission charge rate) or a quantity (of permits). It concentrates instead on the second question noted above: How large should the emission rights underlying the mechanism be, in order to maximise its political acceptability? Could not total rights under a market mechanism be less than the *de facto* total allowed by regulatory standards? Can ownership of the rights be divided among industry and taxpayers? The answers to these questions show the theoretical possibility of using market mechanisms to give a 'win' (i.e. a net economic benefit) to all of three important interest groups in society: environmental users, industry and taxpayers. However, this all assumes perfect information, and a later section of the chapter considers how various types of information costs limit the applicability of this 'win-win-win' result.

The main point of interest in Part I was distribution over time among different generations. Given the inherent complexity of dynamic analysis, we made the simplifying assumption that people at any point in time were homogeneous. In contrast, the main point of interest in Part II is distribution among different interest groups at one point in time. In common with most of the literature on market mechanisms we therefore ignore time, and work in a comparative static framework. We consider differences both among people, which explain the existence of environmental users, industry and taxpayers as separate interest groups, and among firms in an industry, which provide the basic efficiency advantage of market mechanisms as compared to uniform regulatory standards.

CHAPTER 7 (nearly as published in *Canad. J. Econ.*, **25**, 985-91, 1992)

THE SYMMETRY BETWEEN CONTROLLING POLLUTION BY PRICE AND CONTROLLING IT BY QUANTITY*

7.1 INTRODUCTION

The essential result of this Chapter is very simple. Under ideal conditions, controlling excessive pollution or congestion of a scarce public or common property resource by using a price-based instrument such as a fee or charge can be made symmetrical, in terms of short-run efficiency, long-run efficiency,[1] and political acceptability, to using a quantity-based instrument such as a marketable licence or permit. The symmetry between pure charges and sold or auctioned marketable permits has already been shown by Spulber (1985). Here we show that there is also symmetry between charge-subsidies, and corresponding marketable permit schemes where some or all of the permits are freely granted rather than sold. There is thus no *fundamental* reason, as long as the decision has been taken to use some market instrument rather than direct regulation, for choosing control by price instead of control by quantity, or vice versa.

* I am grateful for research support from the Harkness Fellowships, the UK Department of the Environment and the UK Centre for Economic and Environmental Development. I also thank seminar participants at the University of Colorado, Chuck Howe, Gene Mumy, Wallace Oates, Paul Downing and three anonymous referees for helpful comments on earlier drafts. All remaining errors are mine.

1. As usual in the pollution control literature, 'short-run' takes as given the firms that exist in the industry, while 'long-run' allows for the entry and exit of firms.

The key condition for attaining this useful freedom of choice is that, in any given application, both types of instrument embody the same degree of 'environmental ownership,' in the form of symmetrical, private property rights in the resource. However, the relevant literature implicitly or explicitly, but in either case rather inconsistently, rules out this property rights condition for control by price, but does not rule it out for control by quantity. As a result, it often happens that efficient and acceptable instruments are rejected by economists; efficient but unacceptable instruments are proposed instead; while inefficient but acceptable instruments are the ones actually adopted by policymakers. The aim of this chapter is to encourage the adoption of control instruments that are both efficient and acceptable.

In keeping with the existing literature, the argument below uses the language of pollution control, specifically the control of water pollution. However, it can also apply to a range of natural and man-made resources which are not yet privately owned, such as the atmosphere, land for waste dumping, wilderness and wildlife, road space or airport landing slots, where either one-way or mutual (congestion) externalities may arise.

The 'ideal' conditions that are assumed to hold here constitute perfect competition in its fullest sense. We consider a perfectly competitive industry comprising many small firms, each of which is a rational profit-maximiser producing a single output, and discharging a single effluent, emission or waste stream. The effluent is neither storable on the factory site, nor cumulative in the environment, but is continuously assimilated into a well-mixed but finite environmental reservoir. Firms face perfectly competitive markets for their outputs and for their capital and labour inputs, but they own different sets of fixed factors such as enterprise, and therefore have different marginal cost schedules for effluent control. Time-dependent phenomena such as uncertainty and technical innovation in pollution control are ignored. Perfect information is freely available to all firms and to the pollution control authority (hereafter just 'the authority'), and transaction costs are zero; as we shall see in the latter part of Chapter 8, removing this assumption will make a difference to some of the results obtained here, although not to the central insight. Last, but by no means least, a perfect authority, whose sole objective is to maximise public welfare, is assumed.

Contrary to normal practice, we first, in Sections 7.2 and 7.3, state the case for the equivalence of control by price and control by quantity, and then, in Section 7.4, relate the ideas thus raised to the existing literature. Finally, in Section 7.5, we draw out some implications for policy.

7.2 CONTROL BY PRICE: THE CASE FOR THE CHARGE-SUBSIDY

Controlling pollution by price implies the use of charges[2] per unit of effluent added and/or subsidies[3] per unit of effluent reduced. The way in which charges and subsidies can be combined into a 'charge-subsidy' scheme, which achieves short-run efficiency, long-run efficiency, and political acceptability, has been fully spelt out in a neglected paper by Mumy (1980).[4] His scheme is effectively that each polluting firm pays

$$V(E-E_b) \qquad \text{(in say dollars per month)} \tag{7.1}$$

to the authority, where

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2. Also known as fees or taxes.

3. Also known as bribes, payments, or compensation.

4. The name 'charge-subsidy' is mine; Mumy himself referred to 'efficient property rights sharing', to emphasise the property rights involved in the scheme.

V = the charge rate (in say dollars per ton of effluent) set by the authority, which does not vary from firm to firm or with time;

E = the effluent level (in say tons per month). This is under the firm's control and so may vary from firm to firm and over time;[5]

E_b = the baseline effluent right (in tons per month) which is initially given *as a property right* to each existing firm by the authority. E_b may vary from firm to firm but does not vary over time.

If a firm has a positive baseline, and its effluent is less than its baseline ($E<E_b$), it receives a subsidy from the authority. If $E_b=0$ for all firms, the scheme reduces to a pure Pigovian pollution charge. V (which will of course equal the industry's marginal cost of effluent control in equilibrium) is chosen so that the marginal damage cost of the resulting total effluent ΣE is equal to V, thus achieving *short-run efficiency*, given the ideal conditions assumed.[6] ΣE is thus determined on economic grounds, and is not necessarily the same as total baseline effluent ΣE_b, which is determined on political grounds (see below). The scheme therefore may not be revenue-neutral for the authority.

Long-run efficiency is achieved because E_b is a full property right. New firms entering the industry are therefore *not* given effluent rights (so for them, $E_b=0$), while existing firms exiting from the industry *keep* their effluent rights, and receive a subsidy of VE_b in perpetuity. Under these entry-exit rules, the opportunity cost to any firm of producing output Q and

5. Mumy actually considered the more restricted case where effluent is strictly proportional to output and output itself is taxed.

6. Because each firm remains small in relation to the environmental reservoir, the marginal damage cost curve of each firm's effluent is constant. See Burrows (1979) and Collinge and Oates (1982) for the modifications required to the charge scheme if marginal damages increase as the firm's effluent increases.

effluent E rather than closing down production (or not starting production in the first place, in the case of a new firm) is the sum of $C(Q,E)$, the firm's ordinary cost function excluding effluent charges and subsidies; $V(E-E_b)$, the effluent charge-subsidy; and VE_b, the cost of not receiving the perpetual subsidy for closing down. The net opportunity cost to the firm is then

$$C(Q,E) + V(E-E_b) + VE_b = C(Q,E) + VE; \tag{7.2}$$

and since $C(Q,E) + VE$ is the social opportunity cost of production, long-run efficiency is achieved. The baseline effluent right E_b disappears from formula (7.2), so it has no effect on production costs or resource allocation; the invariance proposition of Coase (1960) is thus recovered. Owning E_b effluent rights simply increases the wealth of the firm's owners, and there are no wealth effects because firms are small. Holderness (1979) observed how Coase invariance exists only 'when rights are assigned to closed classes of individuals or entities,' and the above entry-exit assumptions do indeed close the class of owners of effluent rights.

In a charge-subsidy scheme, baseline effluent rights E_b for each firm should be chosen entirely on political grounds (which is why ΣE_b and ΣE may differ). The choice is unlikely to be easy. In many cases *de facto* effluent rights clearly exist in the form of existing effluent standards (Buchanan and Tullock, 1975, 142; Pezzey 1988, 207). However, both environmental and industrial interests often fear, if for quite opposite and incompatible reasons, that formally recognising effluent rights will be disadvantageous to them in the long-term struggle that usually precedes the establishment of any property rights over public or common resources. Whichever is the case, the more quickly and firmly that a formula can be found to settle disagreements between environmental and industrial interests, the sooner and greater will be the economic gain which can then be shared between these interest groups, and also taxpayers and consumers.

7.3 CONTROL BY QUANTITY, AND SYMMETRY WITH CONTROL BY PRICE

The authority can achieve effluent control by a quantity instrument, in a way that is formally symmetrical to the above scheme of control by price, as follows. As with charge-subsidies, the control authority starts by knowing the optimal total effluent ΣE. The authority gives ('grandfathers') each existing firm a free baseline amount E_b of marketable effluent permits (MEPs),[7] and takes such steps as are necessary to create an efficient market to bring together potential buyers and sellers of MEPs. If $\Sigma E_b > \Sigma E$, the authority must then rent back $(\Sigma E_b - \Sigma E)$ permits from the lowest offerer; if $\Sigma E_b < \Sigma E$, it must create an extra $(\Sigma E - \Sigma E_b)$ permits and offer these out for rental to the highest bidder. In either case, the equilibrium rental price of an MEP becomes V, the optimal effluent price.[8] If a firm's effluent $E > E_b$, it legally must rent $(E - E_b)$ permits at a rental price V, whereas if $E < E_b$ it will wish to lease out $(E_b - E)$ spare permits. If the firm closes down $(E = 0)$, it can lease out all E_b spare permits and receive a permanent income of VE_b. As with the charge-subsidy scheme, firms entering the industry do not receive effluent rights (i.e. $E_b = 0$). In all cases a firm producing output Q and effluent E therefore ends up paying $V(E - E_b)$ to the authority, but faces opportunity costs of production equal to $C(Q,E) + VE$. These are the

7. Also known as transferable discharge permits (TDPs), tradeable emission licences, tradeable effluent rights, marketable pollution consents, etc, etc.

8. The talk is of renting rather than selling permits in order to make the symmetry between marketable permits and charge-subsidies more obvious. If the interest rate is r and the permit is permanent, the selling price would be V/r. Other details of this market, for example whether it uses quoted prices or auctions, are not discussed here.

same formulae as (7.1) and (7.2) for the charge-subsidy scheme, so the MEPs achieve the same short and long-run efficiency, and as before, baseline effluent permits can be distributed according to political criteria without impairing efficiency. The whole scheme is presumably similar to that envisaged in a comment on Mumy by Beavis and Walker (1981), though with the important difference that here the total ΣE_b of the effluent baselines does *not* need to be exactly equal to the 'total amount of acceptable discharge,' i.e the optimal total effluent ΣE; if it does, political and economic considerations become entangled again.

The available schemes for control by price and control by quantity are summarised in **Table 7.1**, and under our ideal conditions we have shown that the two types of control are fully symmetrical in terms of efficiency and

Table 7.1 Categorisation of market instruments for effluent control by method of control, and by effluent rights embodied

		Effluent rights owned by firm[1]		
		Zero	Intermediate	Free market level of effluent
Control by price or by quantity?	Price	P1. Pure charge	*P2. Charge-subsidy*[2]	P3. Pure subsidy
	Quantity	Q1. Sold or auctioned MEPs	Q2. Freely granted (grandfathered) MEPs	*Q3. Granted and bought back MEPs*[2]

NOTES

1 The choice of three discrete baseline values reflects what is assumed in the literature, but there is nothing in formula (7.1) which says that effluent rights have to be confined to these values. Chapter 8 explores in detail the implications of allowing effluent rights to take on any intermediate values.

2 Instruments in *italics* are frequently ignored in the literature (for example, by Milliman and Prince 1989).

acceptability. Our key conclusion is therefore that the best control scheme is to formalise the de facto effluent rights of each firm into precise baselines, and then incorporate these baselines as property rights into either charge-subsidy or MEP schemes, with the choice between charge-subsidies and MEPs being determined by practical departures from the ideal conditions.

7.4 ARGUMENTS AGAINST SUBSIDIES

There is little in Sections 7.2 and 7.3 that is technically new, as already noted. However, the symmetry we have established and depicted in Table 7.1 is widely rejected in the literature. It is therefore important to examine this rejection here, before briefly reviewing in Section 7.5 why a fundamentally free choice between control by price and control by quantity is desirable and how it should be made, and suggesting how the debate can move forward.

The literature on effluent charges and subsidies stretches from Kamien, Schwartz and Dolbear (1966) to modern textbooks like Baumol and Oates (1988, Chapter 14).[9] Its essential conclusion is that subsidies are undesirable, for three reasons: one economic, one administrative, and one political. The first, economic reason given is that, in the long run, subsidies encourage excessive entry into a polluting industry, and avoiding this would require the practically and politically impossible task of tracking down potential polluters and subsidising them to stay out of the industry. However, this conclusion entirely depends on the (usually implicit) assumptions that subsidy payments are available to all firms that enter, and

9. An earlier version of this Chapter (Pezzey 1990) contains a more detailed review of this and related literature.

terminated for all firms that exit. The case for these standard 'open-class' entry-exit assumptions, which differ crucially from our 'closed-class' assumptions above, is rarely given. While the standard assumptions may represent the way in which real subsidy schemes generally operate, as noted by Baumol and Oates (1988, 214), there is no theoretical reason why a new firm should not have to buy or rent its effluent rights from existing owners of the environment, just as it must buy or rent its new factory site from existing owners of land.

The second, administrative reason given, for example by Baumol and Oates (1988, 216), is that it would be infeasible to pay subsidies *indefinitely* to firms which have exited. If so, the solution would be the suggestion in Dewees and Sims (1976, 330) that the authority buys out exiting firms' effluent rights by offering lump sum subsidies in compensation (although this could make big demands on the authority's cashflow). The third, political reason is that given by writers such as Spulber (1985, 106), who object to firms owning effluent rights on the grounds that society owns the environment, and recommend pure charging instead. As argued above, this ignores the political reality that many firms have de facto effluent rights and the clout to defend them.

Despite the formal symmetry that we have shown to exist between freely granted MEPs and charge-subsidies under ideal conditions, the former are both much better known and much less likely to be criticised in the literature than the latter; see for example the approval given to granted MEPs in Baumol and Oates (1988, 179). Such writers are much more prepared to accept the notion of environmental property rights with control by quantity than with control by price. As a result, they thus explicitly or implicitly accept the closed-class entry-exit assumptions for control by quantity, and thus ensure that the long-run economic objections of excessive entry to the industry do not arise with freely granted MEPs. Also, MEPs do not get tainted with criticism of related instruments, because of the asymmetric choices of instruments that are made when comparing control by price and control by quantity. For example, Milliman and Prince (1989), in an otherwise comprehensive study of how instrument choice affects technical innovation, choose pure charges and pure subsidies (P1 and P3 in Table 7.1) as instruments which control by price, but sold MEPs and freely granted MEPs (Q1 and Q2) as instruments which control by quantity. Choosing to study pure subsidies instead of the charge-subsidy option (P2) tends to associate control by price in general with the specific moral hazard of pure subsidies, which arises when the level of effluent that firms initially (or hypothetically) discharge in the absence of all regulation is used as the starting point for subsidies. The equivalent objection to MEPs does not arise because no one thinks it sensible even to consider option Q3, whereby firms are given permits equal to what their free market, unregulated discharges would be.

7.5 IMPLICATIONS FOR POLICY

Conventional economic wisdom thus unnecessarily excludes a rights-based charge-subsidy scheme (option P2 in Table 7.1) from serious consideration as a policy instrument. This may have expensive consequences in real cases where pure charging (option P1) is politically unacceptable because of well-established de facto effluent rights, but control by price is more cost-effective than control by quantity. In any given case, practical choices between control by price and control by quantity, and about how much regulation should be retained as a backstop to market instruments, should be based on how well each instrument copes with the way the real world departs from the ideal conditions set out in Section 7.1.

These departures include uncertainty; monitoring and enforcement costs, and how these are distributed between firms and the control authority; storage or accumulation of pollutants; changes over time due to economic growth and technical progress; and vulnerability to monopoly power (see Rose-Ackerman 1977 and Pezzey 1988 for surveys of many of these points). Because of the variety of practical circumstances that can occur, there can be *no general presumption that control by quantity is superior to control by price*.

Uncertainty is worth a special mention. It is well-established, following a seminal contribution of Weitzman (1974) and a recent summary by Baumol and Oates (1988, Chapter 5), that if the authority has good information on the marginal benefits of effluent control, is uncertain about the absolute level of control costs, but is reasonably sure that marginal benefits decrease less steeply than marginal costs increase as effluent is reduced, then control by prices will give greater expected social welfare than control by quantities. Harrison (1983) and Oates, Portney and McGartland (1989) documented cases (concerning aircraft landing noise and urban air pollution, respectively) where these conditions are met, and control by price is economically preferable. In the context of global warming, the choice between carbon taxes and tradeable carbon emission permits may be one where, if effective progress is to be made, using control by price to avoid excessive costs to industry is more important than using control by quantity to achieve precise control over carbon dioxide emissions.

How then can the charge-subsidy idea be added to the menu of instruments considered by policymakers? One way to overcome resistance to the idea may be to change the language used. Kelman's (1981) survey showed that attitudes to effluent charging are greatly influenced by the choice of particular words such as 'fees,' 'charges' or 'taxes'. Clearly, re is also a vast difference in political perception between 'a bribe,' 'a sidy' and 'compensation,' even if all three are financially identical; ich word many writers have chosen to use can hardly be accidental. wever, it is also clear from other policy studies, such as the analysis of US emissions trading scheme in Hahn (1989, 101), that a fundamental ssage of economic analysis – that once a resource has become scarce, eeds to be owned, and priced, if it is to avoid becoming even scarcer – ne that many people do not want to hear, particularly when it is applied he natural environment. The implications of an economic need for the p oceans and the stratosphere to be 'owned' can indeed be disturbing, h practically and psychologically, and may provoke second thoughts ut how far the physical demands of continued economic growth can be wed to proceed. However, while they do proceed, there is an urgent d to find ways of controlling resource use that are both efficient and eptable. The delicate task of promoting schemes which contain the essary elements of subsidy and effluent rights, whilst trying to avoid ect use of such emotive words, is therefore one which economists should duck.

CHAPTER 8

ON THE POLITICAL AND INFORMATIONAL ECONOMY OF DISTRIBUTING THE EFFICIENCY GAINS ROM MARKET MECHANISMS OF POLLUTION CONTROL*

l INTRODUCTION

Progress around the world towards the use of market mechanisms of llution control, such as emission charges and marketable emission permits lEPs),[1] generally remains slow (Hahn and Stavins 1992). This is despite me promising developments in recent years, and despite the well-known

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* I am greatly indebted to Bob Kohn for providing the original stimulus to this per, and for numerous discussions about its earlier versions. I also thank Mick mmon, Linda Critchfield, Tony Leiman, Geoff Lewis, Danny McCoy, Renee co, Charles Perrings, Joe Swierzbinski and Tom Tietenberg for helpful comments. l errors and omissions are mine. I am grateful for research funding under award . L320-27-3002 of the U.K. Economic and Social Research Council's Global vironmental Change Research Initiative.

[1] The word 'pollution' is ambiguous: it can mean the amount of pollutant emitted, concentration in the environment, the physical damage it does to the environment, the value of this damage; and each of these four quantities can be measured ative to zero emissions, or relative to some acceptable level of emissions. henever clarity is needed, I avoid 'pollution' and use 'emission' or 'emissions' to an the first quantity, measured relative to zero. When 'pollution' is used, as in general phrase like 'pollution control policy', the ambiguity will be unimportant. other semantic point is that my analysis applies to any medium, so I could have d the language of water rather than air pollution and talked of 'effluent' rather n 'emissions'. Finally, I use the word 'control' throughout, rather than its onyms 'abate' or 'abatement'.

ort run theoretical efficiency of these mechanisms when used to control 1issions from a heterogeneous industry (Baumol and Oates 1971, ontgomery 1972). One possible reason is that proponents of market echanisms still pay too little attention to the following dicta:

> "...economists who continue to support the penalty tax alternative [i.e. Pigovian taxation of pollution rather than direct regulation] ... had best ...begin to search out and invent institutional arrangements that will make the penalty tax acceptable to those who are primarily affected". (Buchanan and Tullock 1975, p147)

> "...the probability that a policy is adopted depends on who gains from it, who loses, and by how much." (Zeckhauser 1981, p215)

ecent policy analyses by Pezzey (1988), Hahn (1989) and Hanley, Moffatt d Hallett (1990) also stressed that the distributional effects of proposed arket mechanisms are crucial in determining whether or not they are lopted. However, these papers focused mainly on the two types of market echanism which have seen practical use, namely redistributive emission arges in Europe, where a low charge rate is aimed at raising revenue for bsidising emission control investments, and freely-granted grandfathered') MEPs in America.

Here I describe a more general design of market mechanism which allows e policymaker, at least in a world of perfect information, a much wider, ntinuous range of policy options. The key features of the mechanism are e, it exists in both charge ('control by price') and marketable permit control by quantity') form. Two, it can always be 'win-win-win', that is, oduce a gain for each of three interest groups: industry, environmental ers, and taxpayers. Moreover, these two features are available without y sacrifice of short or long run efficiency. The 'win-win-win' property achieved by setting industry's total emission rights at some level between ro and a level of emissions less than the total level permitted by the rrent, inefficient system of emission control. Such a property should, one ɔpes, improve the political acceptability (and therefore actual progress) of arket mechanisms, although the open recognition of industry's emission ghts that is required would need careful handling.

Section 8.2 describes the win-win-win mechanism diagrammatically, and ɔserves that, following the result of Chapter 7, the mechanism can eoretically be implemented by either charge-subsidies or partly-sold MEPs. ıbsequent sections spell out subsidiary results, hidden assumptions and ecessary qualifications to this basic result. Section 8.3 discusses the choice interest groups and control mechanisms made in Section 8.2, and shows at the literature has been quite confined in its choices up till now. Section 4 shows that, unless firms' emission rights are changed in an unlikely, ıequal way, one cannot extend strict Pareto improvement to the firm level, ıd 'average' firms may well lose from a win-win or win-win-win echanism. Section 8.5 argues that it would be unusual for a pure market echanism (one with zero emission rights for industry) to have the win-win-in property, since this would imply that the existing or any feasible ternative regulatory system of control must be extremely inefficient. ection 8.6 discusses several important ways in which various types of formation costs may limit the practicality of the win-win-win mechanism. inally, as an illustration of the paper's arguments, Section 8.7 shows how e recent SO_2 emissions trading program in the USA could fairly easily ıve been an example of this win-win-win mechanism, but in fact is only win-win mechanism. Section 8.8 concludes.

2 BASIC DESCRIPTION OF THE WIN-WIN-WIN MECHANISM

The analysis is confined to an industry which discharges a single, non-
›rable, non-cumulative pollutant into a well-mixed environment.[2] Firms the industry face perfectly competitive markets for their outputs and for ›ir capital and labour inputs, but they own different sets of fixed factors ch as enterprise, and therefore have different marginal cost schedules for luent control.[3] We abstract from time-dependent phenomena such as certainty about the future, and technical innovation in pollution control; latter omission therefore ignores the possibly important dynamic iciency benefits that can flow from market mechanisms (Downing and hite 1986, Milliman and Prince 1989). *For the moment*, all information d transaction costs are assumed to be zero; this is relaxed in Section 8.6. stly, we assume that the pollution control authority (hereafter 'the thority') aims to maximise public welfare, has no operating costs, forces any control system perfectly, and passes any revenue raised by a ırket mechanism directly to taxpayers. The important complications of ıltiple pollutants local variations in environmental quality, imperfect mpetition and realistic authorities are all left for further work.

Suppose that such an industry has the structure of emission control costs ›wn in **Figure 8.1**. There is a continuous, monotonic, marginal control

[2] But following Kohn (1994), I do not need to assume that each firm faces a izontal curve for the marginal damage caused by its emissions.

[3] As shown in more detail later, without the assumption of heterogeneous control ts, it is hard to see why the existing regulatory system should be inefficient, and ıce where the efficiency savings from any market mechanism are to come from. vertheless, plenty of literature on market mechanisms assumes identical firms and ıce uniform control costs (see for example Buchanan and Tullock 1975, Spulber 35, and Kohn 1985, 1994). While this simplification improves tractability, it can sight of the main practical reason for being interested in market mechanisms.

›st curve SA if control is by some form of inefficient, direct regulation.[4] lower curve SGFC would apply if control by an alternative, more 'ficient system. For the sake of exposition, we assume henceforth that this ternative is the kind of market mechanism (discussed further below) which :hieves *fully efficient* control: i.e., control that is efficient in both the short ın, given the existing number of firms, and in the long run, allowing for ıtry and exit of firms. However, most of our results also apply to any ıift from inefficient to more efficient, even if not fully efficient, control.

With either inefficient or fully efficient control, the industry would emit flow OS of pollutant if uncontrolled. Marginal damage costs are not ›ecified precisely, and could for example be either of the dashed curves FA · CBA. A change from inefficient control at A to fully efficient control at would lower industry's total control costs by area GAS, while leaving its tal emissions unchanged at ON. Market mechanisms exist which could :hieve this change in a *revenue-neutral* way. For example, the authority ›uld issue free ('grandfathered') MEPs to all firms equal to their current nissions under standards, which total ON. This would indirectly establish uniform price OP per unit of emissions, and hence achieve the efficiency ıins GAS while leaving these gains as a clear 'win' for industry, to be alised by permit trading among firms.

However, the authority could instead achieve a 'win-win' outcome which nefits environmental users as well as industry, by establishing an emission ice of OP′ rather than OP. This would move the industry from G to F, ıs lowering total emissions to OM, which would benefit environmental

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[4] The assumption of monotonicity is standard, and is supported by much empirical idence. However, economies of scale in the control cost function are theoretically ı practically possible, and can cause considerable complications (Pezzey 1988, pp. ›-241; Harford 1989).

users (by an amount MFAN or MBAN, depending on whether FA or CBA is the marginal damage curve). To continue the MEP example, the authority could issue MEPs totalling OM instead of ON, by giving each firm permits equal to OM/ON times its current regulatory standard. The overall move from A to F would still give industry a net benefit since, *with the curves as drawn*, MFGN $<$ GAS; that is, the extra cost of reducing total emissions from ON to OM is less than the savings in keeping total emissions at their original level ON. Whatever the origin of the win-win idea (it is implicit in Pezzey 1988, pp. 214-6, but doubtless occurred to others much earlier), it has been little discussed in environmental economics articles or major textbooks, probably because of its political rather than economic nature. Yet, as noted in Section 8.7, it can play a vital role in securing progress towards more efficient pollution control.

The authority could also go beyond revenue-neutrality, and achieve a 'win-win-win' outcome which benefits taxpayers as well as industry and environmental users. Continuing our MEP example, it could give firms permits totalling less than OM, say OL, and sell the remainder LM to them at the competitive price OP′. Taxpayers would then receive the sales revenue LDFM,[5] but the move from A to D would still be a net 'win' for industry since, again with the curves as drawn, LDFGN (the revenue paid

[5] The revenue, rather than being given to taxpayers as lump sums, could be used to reduce other taxes in the economy, such as income taxes. Stemming from Terkla (1984), there is a widely-held view these days (Pearce 1991, Repetto 1992) that the latter use would give a further efficiency gain to the economy, by reducing distortionary deadweight losses. However, this view is not without critics (Bovenberg and de Mooij 1994) who see it as too simplistic. Whichever way this debate is resolved, the revenue *per se* will be seen as a benefit by taxpayers, even if the political cost of raising it may sometimes be felt to be too high, as noted in Section 8.7 in the context of the USA's SO_2 trading program.

to the authority under the market mechanism, plus the extra control cost) is less than GAS (the control cost saving on the original emissions). The central observation of this paper is that, for any sensible position of the three control or damage cost curves,[6] one can *always* reach such a point D with the 'win-win-win' property, by a suitable choice of emission price OP′ and free permit total OL.

Moreover, this result is not confined to MEPs ('control by quantity'). It holds for the more general market mechanism defined in Chapter 7, under which each firm has to pay the authority an amount in respect of its emissions equal to

$$V(E-E_b) \quad \text{(in, say, dollars per month)} \qquad (8.1)$$

where V is the unit emission price, E is the firm's actual emission level, and E_b is called its *baseline emission right* (or just 'baseline'). MEPs are one form of this mechanism, with V being the market price at which total permit demand equals the total ΣE issued by the authority, of which ΣE_b are given away free (where the summations are across all firms in the industry). Another form of (1) is a system of *charge-subsidies* ('control by price'), with V being the emission charge set by the authority, ΣE the industry's total emission demand in response to this, and ΣE_b the sum of the baselines which form the boundary between charge and subsidy for each firm. I showed in Chapter 7 that as long as the baselines in either form are treated as full property rights, the mechanism is efficient in the long as well as the

[6] Even assuming linear marginal control and damage cost curves, there is still enormous variety in the way that the three curves in Figure 8.1 can be drawn (since there are 5 independent parameters determining their position), and a corresponding range of relative sizes of the various areas under the curves.

short run.[7] Freely given MEPs and charge-subsidies which use identical distributions of baselines (which represent lump-sum transfers of wealth) among firms are then economically and politically equivalent control systems, at this level of theoretical abstraction.

To use the general mechanism (1) to reach a win-win-win point like D in Figure 8.1, the authority would set

Emission price $V = \mathrm{OP}' > \mathrm{OP}$ (which defines points M and F) (8.2)

Baseline total $\Sigma E_b = \mathrm{OL} < \mathrm{OM} < \mathrm{ON}$ (which defines L and D) (8.3)

in such a way that LDFGN < GAS. Note that rule (3) treats the baseline total ΣE_b as a continuous rather than a discrete variable. MEPs must be *partly*-sold rather than all freely-given; or equivalently, charge-subsidies must be *positive*- rather than zero-revenue. Rules (2) and (3) allow a range of choices of D, and any particular choice determines how the efficiency benefits of market-based emission control are divided among the three major interest groups. I suggest that this division can and should be deliberately used to make the shift to market mechanisms as politically attractive as possible.

8.3 THE CHOICE OF INTEREST GROUPS AND CONTROL MECHANISMS

The choices of interest groups and market mechanisms made in the above analysis are important in determining its relevance to real policy issues. It is thus worth discussing these choices further, and comparing them to other choices made in the literature. Regarding interest groups, I have distinguished industry, taxpayers and environmental users because they do

[7] In particular, charge-subsidies that treat E_b as a property right do *not* cause excessive entry or insufficient exit of polluting firms in the long run.

exist as separate, powerful lobbying groups in almost all industrial countries. This separation ultimately reflects *differences among people*: differences in their ownership of individual firms and of industry overall, in their payment of taxes, and in their use of the environment. To see why, recall that if the authority sold marketable emission permits (MEPs) to firms, the sales revenue would (under our assumption about the authority's goals) go to taxpayers. If instead the authority gave free MEPs to firms, the same revenue would remain as part of firms' profits. But if everyone paid the same taxes and owned an equal share of each and every firm, they would be indifferent to this shift from sold to freely given MEPs, because it would leave their after-tax incomes exactly the same. In practice, differences in ownership make people far from indifferent to such a change, and liable to form special interest groups which lobby for or against it.

We will also focus in Section 8.4 on different types of firm within an industry.[8] This is because, as already pointed out in footnote 3, without variation among firms, it is difficult to see why the existing regulatory system of pollution control would be inefficient. Also, different types of firm, and often individual firms, clearly do play major lobbying roles in public debates about pollution policy, and our analysis helps to explain why. But we generally ignore different types of taxpayers or of environmental users, because, given the millions of people who usually pay taxes and use any given environment, it is much harder to organise *sub*groups of them into effective lobbying forces. However, if there are just a few sharp divisions among taxpayers or environmental users, assuming that the interests in each

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[8] But we ignore another desirable distinction to make within industry, which is among the interests of labour, management and ownership. Given the variable degree to which these groups have sunk their capital into their particular firm or industry, they may have significantly different interests in environmental policy.

of these two groups are uniform could obviously become misleading; but this is another complication left for further work (see for example Pearce 1980 on the differences in pollution damages suffered by rich and poor environmental users).

Regarding the choice of instruments, the crucial innovation above, which is implicit in equation (1) but was undeveloped in Chapter 7, is to treat one dimension of the choice, the level of baseline emission rights, as a continuous rather than a discrete variable. As illustrated in **Table 8.1**, which summarises the interest groups and market mechanisms chosen by Dewees (1983), Mestelman (1985), Chapter 5 of Tietenberg (1985), Hahn (1990) and Chapter 7 of this thesis, the choice of baseline in the existing literature on the political economy of pollution control has until now been confined to at most three levels for each firm. $E_b = 0$ corresponds to a *pure* market mechanism (a pure charge or sold MEP), E_b = current emissions corresponds to a revenue-neutral mechanism (a zero-revenue charge-subsidy, or a freely given MEP), and E_b = unregulated emissions corresponds to a pure subsidy, or an over-given and bought-back MEP. The purpose of the Table is to highlight the rather limited choice of interest groups and mechanisms that has so far been made. In particular, while most authors nowadays recognise the difference between sold and free MEPs, charge-subsidies continue to be widely ignored (for example by Rajah and Smith 1993 and Howe 1994) as an alternative to pure charges.

8.4 THE PROBLEM OF THE AVERAGE FIRM

Unfortunately, it is impossible to find a realistic design of a win-win or win-win-win market mechanism which also benefits every firm in the industry. 'Average' firms tend to lose out, which somewhat mars the

Table 8.1 Choices of interest groups and market mechanisms of pollution control made by papers which study the political economy of market mechanisms

<table>
<tr><th colspan="3">Reference
Topics covered</th><th>Dewees (1983)</th><th>Mestel-man (1985)</th><th>Tieten-berg (1985)</th><th>Hahn (1990)</th><th>Chapter 7, this thesis</th></tr>
<tr><td colspan="2" rowspan="5">Political interest groups</td><td>Industry – capital</td><td>✓</td><td rowspan="2"></td><td rowspan="2">✓</td><td rowspan="2">✓</td><td rowspan="2">✓</td></tr>
<tr><td>Industry – labour</td><td>✓</td></tr>
<tr><td>Resource owners</td><td></td><td>✓</td><td></td><td></td><td></td></tr>
<tr><td>Taxpayers</td><td>(✓)[1]</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Environmental users</td><td>(✓)[1]</td><td></td><td></td><td>✓</td><td></td></tr>
<tr><td rowspan="6">Market mechanisms</td><td rowspan="3">Control by price</td><td>Pure charges</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Charge-subsidies</td><td></td><td></td><td></td><td></td><td>✓</td></tr>
<tr><td>Pure subsidies</td><td></td><td>✓</td><td></td><td></td><td>✓</td></tr>
<tr><td rowspan="3">Control by quantity</td><td>Sold or auctioned MEPs[2]</td><td>✓</td><td></td><td>✓</td><td rowspan="2">✓[3]</td><td>✓</td></tr>
<tr><td>Freely given MEPs</td><td>✓</td><td></td><td>✓</td><td>✓</td></tr>
<tr><td>Given and bought back MEPs</td><td></td><td></td><td>✓</td><td></td><td>✓</td></tr>
</table>

NOTES

1. While Dewees calculates the effects of different instruments on tax revenues and emission levels, he does not consider the political reactions to these effects.
2. MEPs = marketable emission permits.
3. Hahn does not specify whether his 'marketable permits' are sold or free.

mechanisms' appeal. To see this, suppose that the competitive industry in Figure 8.1 comprises an equal number of three types of firms, with the marginal costs of controlling emissions being as shown in **Figure 8.2** for each firm, where the horizontal scale has been expanded. One type has a High marginal cost curve *sea* of controlling emissions below level O*s*, one has a Low curve *sgc*, and one has the 'Average' curve *sd*.[9] We could consider many firms of different sizes and control cost curves, but there is little loss in generality in working here with just three firm types (although there is a loss in generality in assuming *linear* marginal control costs: see footnote 13).

Assume that the pre-existing system of control imposes a uniform regulatory standard of O*n* per firm.[10] To achieve the same emissions total, a revenue-neutral 'win' market mechanism (either a zero-revenue charge-subsidy, or a freely given MEP) would have to create an emission price OP, and would typically choose to give baseline emission rights of E_b=O*n* per firm. Low would then emit O*h* and thus spend *hcgn* on extra control costs relative to the uniform standard, but gain *hcdn* (either from the subsidy or from selling *hn* spare permits to High). It would thus enjoy a net benefit of

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[9] Geometrically, point *d* is defined by P*d*=(P*c*+P*e*)/2 rather than by *nd*=(*ng*+*na*)/2, so the Average firm defined by *sd* has average *emissions* when the market mechanism is applied, rather than average *control costs* when uniform standards are applied. Note that for a firm's 'control cost curve' to be independent of pollution control policy, the firm must be free to equalise the marginal control costs of all its sources of a given well-mixed pollutant. However, many existing direct regulations prescribe emission limits for particular sources within a firm with little regard to control costs. This can cause great inefficiency at the firm level (Maloney and Yandle 1984), and a correspondingly great opportunity for more flexible regulation such as 'bubbling' to enable a multi-source firm to achieve lower control costs. But I would not regard this as an example of savings from a market mechanism, since I use the term to mean only a system which enables reallocation of control costs among *different* firms (either directly with each other, or via the authority).

[10] SA, the industry-level control cost curve under regulation in Figure 8.1, would then be the vertical average of the firm-level curves in Figure 8.2, multiplied horizontally by the number of firms; while SGFC would be the horizontal sum of the firm-level curves.

cdg, while High would get a corresponding net benefit of *dae*. The switch from uniform standards to this market mechanism would have no effect on environmental users (since total emissions are unchanged) or on taxpayers (because of the revenue-neutrality). So Low and High have benefits which can offset the net cost to them of the further step to a 'win-win' or 'win-win-win' mechanism, assuming that this step entails not just a higher emission price but also a lower emission baseline for each firm.

But under the 'win' mechanism, Average's emission is unchanged at O*n*, which leaves it no net benefit to offset any further cost. And it seems inevitable that there would be a cost, since for Average to receive a *higher* baseline from a win-win or win-win-win mechanism while Low and High receive lower baselines (as some firms must, since the baseline total ΣE_b must be lower) would seem politically implausible, even though it would not harm economic efficiency and could still leave every firm in the industry better off. So an average or near-average firm is very likely to be worse off under a win-win or win-win-win mechanism, and thus to oppose it. Moreover, it may be easy for such opposition to outweigh Low's and High's support politically, since losses are often valued much more highly than financially equivalent gains (Knetsch 1990). The fewer near-average firms there are, the greater should therefore be the chance of such mechanisms being adopted. Whether or not this prediction is borne out in practice remains to be seen; probably no one has yet looked for any data that could confirm or deny it.

Also, average firms may be less of a problem than would appear from Figure 8.2, because their definition is actually rather fluid. For example, in **Figure 8.3**, firms A, B and C have marginal control cost curves A′A″, B′B″ and C′C″ respectively. At emission price OP_1, Firm B is the average firm of the three because its emissions are the average of C's and A's. But by the same reasoning, because of the different curvatures of A′A″, B′B″ and C′C″, as the price rises to OP_2, OP_3 and then to OP_4, which firm is the average one changes from B to A, to C, and then back to firm B again. So unless the future emission price under a market mechanism is clear, it may be hard to identify near-average firms.

8.5 CAN A PURE MARKET MECHANISM BE WIN-WIN-WIN?

The overwhelming political disadvantage of a pure market mechanism (a pure charge or fully sold permit) is that it makes firms pay transfers to the authority. The sum of these transfers could easily exceed the industry's overall cost saving (as certainly is the case in Figure 8.1, where OPGN $\gg$ GAS), and this will ensure industry's opposition. However, one can construct contrary, special cases like **Figure 8.4**, where both firm and industry-level cost curves have been drawn for a two-firm industry. Control costs diverge so sharply here between the High and Low cost firms that a move from uniform standards to a pure market mechanism will benefit the industry as a whole, since here GAS exceeds OPGN.[11]

However again, the distribution of this overall benefit between the two firms is very uneven: High saves ($dae-\mathrm{OP}dn$), whereas Low spends

[11] If all the firms' marginal cost curves are linear, the conditions for this to happen are straightforward to calculate. If the existing degree of overall emission control, NS/OS$=:C$, and the ratio of industry control costs under uniform standards to costs under efficient control, NA/NG$=:M$, then $C(M+1)>2$ means that a pure market mechanism will benefit the industry. One can further calculate M from the distribution of marginal control costs among firms, if these are known. For example, if there are equal numbers of two types of firm with marginal costs in the ratio $1:m$ ($m>1$), then it can be shown that $M = (m+1)^2/4m$. If however firms' costs are distributed uniformly between 1 and m, then $M = [(m+1)\ln(m)]/2(m-1)$.

OP*cgn*. Indeed, changing from uniform standards to a pure mechanism would impose a net cost on any firm whose emissions in response to the price OP would be below the standard O*n*, since such a firm would pay higher control costs under the mechanism, and also charges or permit costs for its remaining emissions. This clear division into winners and losers would make a pure mechanism politically much less attractive, even if it could ever actually lower industry's total financial cost of emission control.[12]

Also, if control costs were as hugely divergent as in Figure 8.4, it is more likely that the authority would have known this. It could then have applied (and if not, still could apply) more *flexible regulation* than uniform standards, such as the British policy of requiring the use of the 'best available technology not entailing excessive cost' (BATNEEC) to control pollution. In Figure 8.4, an example of flexible regulation would be that the authority sets standards of O*j* (=(O*h*+O*n*)/2) for Low, and O*q* (=(O*n*+O*r*)/2) for High. This would reduce the cost of reducing total emissions from OS to ON under standards by *nabq*−*jfgn*, whilst still leaving Low paying *jfs* overall, which is much less than the *qbs* that High would pay. When compared to flexible regulation, such as any standard between O*h* and O*n* for Low and between O*n* and O*r* for High, a pure market mechanism may thus no longer give a net benefit to industry, even in this special case.

[12] An administratively pure mechanism might however be achievable if existing emissions are low enough for the authority to be able to 'buy them out' by offering lump sum compensation (as suggested by Dewees and Sims 1976, p330) when introducing a pure mechanism. This might be worthwhile to the authority because it would eliminate the cost of administering positive baselines, which is one of the information costs discussed in Section 8.6 below.

8.6 INFORMATIONAL CONSTRAINTS ON MARKET MECHANISMS

We now relax the assumption of perfect information and certainty, and recognise the existence of various types of information costs and uncertainties regarding actual emissions, emission control costs, and emission damage costs. Some such recognition is logically unavoidable, since various information costs played a major role in creating both the existence of pollution regulations (rather than reliance on Coaseian bargaining), and their inefficiency (which market mechanisms seek to improve upon). It is also a recognition forced upon us by case studies of the real world applicability of market mechanisms (see Hanley and Moffatt 1993 for a recent review and a new case study). We assume throughout this section that all information costs are independent of the actual level of emissions, so that the same diagrams as before can be used for marginal analysis. However, we now have to remember that there may be hidden, discrete jumps in information costs as we switch from one control system to another. The globally optimal system may therefore not be at the marginal optimum suggested by diagrammatic analysis.

Even if policymakers may originally have wished to create a regulatory system which was economically efficient at the margin, information costs would have greatly hampered them. For an authority would have needed to know each firm's control cost curve in order to calculate efficient standards, which in Figure 8.2 would be Oh, On and Or for the Low, Average and High firm respectively, where $Oh+Or=2On$. The cost of this information might well have been prohibitive, as recognised long ago by Kneese, Ayres and d'Arge (1970, p. 96), if not earlier. Even if it had not been, such firm-specific standards would probably have been politically unacceptable. Firms would have perceived them as 'unequal treatment', since (again in Figure 8.2) the area *hcs* that the Low firm would pay in control costs would be much more, with the curves as drawn, than the *res* that the High firm would pay.[13] Transferring a lump sum of *nder* (=*hcdn*) from the High to the Low firm could solve this problem, but only if the authority has the political will to combine financial transfers with a regulatory system, as well as the information necessary to calculate such transfers correctly.

So either information costs or unresolved equity problems typically ruled out, and would continue to rule out, efficient standards. The alternative efficient system, namely market mechanisms, was also ruled out at the time, partly due to ignorance or ideological resistance to the idea of treating the environment as a commodity (Oates 1994), partly due to scepticism about the information costs of making a market mechanism work, and partly due to existing polluters' self-interest in retaining standards (Buchanan and Tullock, 1975). Control systems therefore were based on inefficient standards (although, as noted above, not necessarily uniform standards). This reality, together with reduced ignorance of and ideological resistance to market mechanisms over the last twenty years or so, provides a clearer rationale for wanting to introduce market mechanisms than some writers such as Baron (1985, p229) have allowed for.[14] But the reality of

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[13] The caveat "with the curves as drawn" is needed because if *sgc* is sharply convex to the origin instead of linear, area *hcs* could be smaller than area *res*. Such an outcome seems unlikely, but only measurement of individual firm's control cost curves could confirm this.

[14] Closer attention to the extensive literature on regulation under asymmetric information (such as that stemming for example from Baron and Myerson 1982) would pay some rewards here, but this is another topic left for further work.

information costs also forces us to recognise three important qualifications to our findings about market mechanisms.

Firstly, control by price (charge-subsidies) and control by quantity (MEPs) will no longer generally be equivalent means of implementing the win-win-win mechanism. For as is well known (Baumol and Oates 1988, Chapter 5; Tisato 1994), uncertainty about the location of the industry's marginal control cost curve generally means that either control by price is superior to control by quantity, or vice versa.

Secondly, information costs may make it politically impossible for either a win-win or a win-win-win mechanism to reach marginal social optimality of the conventional, *potential* Pareto kind (not that I claimed such optimality as a virtue of the mechanisms in the first place, though). Consider Figure 8.1 again. If the marginal damage cost curve is the dashed line FA, both the win-win mechanism at F and the win-win-win mechanism at D are also socially optimal at the margin. But if the damage curve is actually CBA, a move to the marginally optimal point C would impose a net cost on industry, since here KCGN > GAS. One could in theory avoid this by taxing away part of the environmental users' total benefit KCAN and transferring it to industry, still leaving environmental users with a 'win' overall. However, unless environmental users are either remarkably few in number or remarkably homogeneous, it could well be prohibitively expensive to estimate both the total benefit and its distribution among possibly millions of environmental users. And without such information, taxing the benefit in order to move to the potentially Pareto-optimal emission level OK would often be politically impossible.

Thirdly, even for the move to the win-win-win point D, any formal emission rights will still incur a more or less fixed cost to administer (as already noted in footnote 12), both initially to distribute rights, and thereafter to enforce them. Whether or not this fixed information cost is outweighed by the efficiency savings GAS will vary from case to case. For example, emission rights are likely to be vastly more expensive to administer in an industry with millions of separate small polluters, such as road transport, than in a concentrated industry such as electricity generation from fossil fuels.

8.7 THE EXAMPLE OF THE USA'S SO_2 TRADING PROGRAM

The above analysis is applied here to the recently started program to use marketable allowances for SO_2 control in the USA, as described by Rico (1993). The 1990 Amendments to the Clean Air Act set a limit for total SO_2 emissions in the USA at about 60% of 1980 levels, to be achieved by 2010 using a program of transferable SO_2 'allowances' (which are not quite marketable emission permits, as discussed below) in two phases, starting in 1995 and 2000. Only electric utilities, which account for about 70% of US emissions of SO_2, are compulsorily included in the program. They are given free allowances in advance, for 1995 and thereafter, at a level based on about half of their historic (1980) emissions. New utility boilers are not allocated allowances, but must buy them from existing utilities. Industrial sources can ignore the program, but if they wish they can opt into it, be given allowances based on all of their historic emissions, reduce their emissions, and then sell their spare allowances to a utility. Each year about 3% of existing allowances are compulsorily put on open sale (mainly at auction, but some at fixed prices) by the authority (the E.P.A.), which then returns the proceeds to the utilities from whom the allowances are requisitioned. The purpose of this is to lubricate the allowance market in two ways: by providing a guaranteed source of allowances for new utilities that need them, in case existing utilities try to hoard their allowances; and for the early auctions, by providing a signal on allowance prices to the market. The allowance market is nationwide, but subject to local limits which stop pollution 'hotspots' developing.

Ignoring the dynamic and geographic aspects of the scheme and of the acid rain damage it is designed to curb, I suggest the following stationary, qualitative interpretation of it using **Figure 8.5**. One can suppose that the industry marginal control cost curve is SHA under standards, and that the policymakers' perception used to be that marginal damages from total SO_2 emissions were QH. Optimal emissions under standards were thus OR, with associated total control costs of RHS. New scientific information on acid rain damage then shifted the perception of the marginal damage curve up to something like FA or CA (the precise slope does not matter). To respond to this by moving to A and cutting the standards total to ON, the new optimum under standards, would make utilities incur extra total control costs of NAHR.

Instead of this, the new SO_2 allowance program can be seen as creating and giving away OM marketable allowances, where OM happens to equal about half of utilities' total emissions in 1980. This lowers the utilities' overall control cost curve from SHA to the efficient curve SJGFC, and their total emissions to OM. Their extra control costs are then (MFJR−JHS), which on Figure 8.5 happens to be much lower than the NAHR incurred under standards. Environmental users are better off because emissions fall by MN. The scheme is thus a revenue-neutral, win-win market mechanism, because both utilities and environmental users gain. According to Rico (1993), this was crucial in gaining acceptance for the scheme: "This political calculus ensured environmental support for the stringent emissions goal, and economic support for a sizable test of economic principles to reduce the costs of environmental control." It also happens to conform to the Standard Polluter Pays Principle, as defined by Pezzey (1988, p208), although Rico did not mention this as an advantage.

The scheme is not win-win-win, because taxpayers gain nothing, but in theory they could have. If the E.P.A. had given away only an intermediate amount OL of allowances, and auctioned off the remainder LM, then utilities would be paying the E.P.A. (and hence taxpayers) a flow of LDFM in revenue. Yet, since GAS > LDFGN in the Figure as drawn, utilities would still benefit from the market mechanism, compared to how much tighter standards at A would have collectively cost them. The benefit to environmental users would remain unchanged, giving a win-win-win mechanism overall. One obvious way to have achieved this would simply have been for the E.P.A. to pass the revenue raised from the 3% annual forced sale of allowances on to taxpayers, rather than return it to the utilities.

However, Rico (1994) suggested that proposing to raise revenue from the program would have been politically impossible, even when the original legislation was being drafted, because of the political perception that it would be unfair to force the utilities to pay a new tax as well as reduce their emissions. To me, this suggests in turn one of three possibilities. Either (i) the curves were such that MFGN > GAS (unlike in Figure 8.5), so that the utilities as a group were already going to lose out as a whole from the program, and were thus in a strong position to resist any transfer to taxpayers; or (ii) the utilities would benefit from the program, but many influential 'average' utilities would not, and mounted a powerful opposition to any transfers; or (iii) almost all utilities would benefit from the market flexibility which the program gives them, but nevertheless they mounted a powerful public relations campaign to give the impression that they would not, thus deterring any transfers. One can only speculate which of these was true, and whether or not it would have remained true if the more expensive alternative of tightening standards had been seriously proposed as the alternative to a win-win-win distribution. But it does show that, even in a country with such a huge budget deficit, a 'win' for the taxpayer is still a fairly weak player in the political poker game.

As flagged above, SO_2 allowances do not quite correspond to theoretical marketable emission permits, since the legislation specifically states that the allowances are not property rights, continuing a trend noted by Hahn (1989, p101). For example, opted-in industrial firms can sell spare allowances only if they have maintained the same output of their product; and emission reductions as a result of reduced output (or even a complete shutdown) are not considered saleable. According to Rico (1994), the main reason why allowances are not property rights is because of uncertainty about whether the 40% reduction in total emissions by 2010 will achieve its stated environmental goal. If it does not, and allowances were property rights, the state would then need to spend large amounts of money buying them back in order to protect the environment. The economic inefficiencies created by allowances not being full property rights were considered to be a price worth paying to avoid the risk of such a future drain on state revenues. However, I suggest that this understandable difficulty could perhaps be overcome in a more efficient way, by mimicking the commercial property market and creating emission rights as long leases, rather than as freeholds in perpetuity.

8.8 CONCLUSIONS

Political will can sometimes push through economic reform against the opposition of powerful interest groups who are harmed by it. But if reform can be redesigned so that powerful groups are not harmed, without sacrificing the very efficiency gains that it is to meant to achieve, then it is surely more likely to happen. My main conclusion here is that, at least in a simple competitive model with perfect information, market mechanisms of pollution control can be redesigned in such a way. A win-win-win outcome is possible, with taxpayers, a polluting industry, and the users of the environment it pollutes, all getting a positive slice of the full efficiency gains of market-based control.

The key to retaining full (i.e. long as well as short run) efficiency is to treat the emission baselines that are embodied in any market mechanism as full property rights. Firms entering the industry should receive no baselines, and firms leaving the industry should not have their baselines confiscated. Such ownership of the environment may cause uncomfortable reflections on the scale of human encroachment on the planet's once vast wildernesses, and so should probably be downplayed as far as possible in the political process. Given, however, that such encroachment has occurred, environmental ownership is necessary to prevent excessive industry growth, and to allow charge-subsidies to achieve long run efficiency and thus be a useful alternative to marketable emission permits.

The key to achieving a win-win-win division of the efficiency gains is to accept that, to be politically realistic, emission baselines under a market mechanism can rarely if ever be zero, but can be set at less than current emissions. Setting baselines so that the efficiency gains of market-based control are divided between environmental users and industry is already at the heart of the USA's recent SO_2 trading program, and such a win-win approach to policymaking is clearly fruitful. Less recognised, and the central message of this paper, is that a three- rather than two-fold division of these gains is possible, and perhaps desirable: taxpayers can receive some but not all of industry's share of the efficiency gains, without harming marginal efficiency. With marketable permits, the state could give away a majority of the permits issued, sell the remainder (thus raising revenue for reducing taxes or the government deficit), and yet still leave industry better off. Equivalently, with charge-subsidies, the baselines which divide charges from subsidies could be set low enough to yield a small revenue for the taxpayer, but high enough again to leave industry better off.

However, these results about the win-win-win mechanism are in practice subject to a number of political and informational limitations. Unless baselines are redistributed in an unequal way, the mechanism cannot guarantee net gains for all firms. If the control authority has poor information about control costs, either the charge-subsidy form of the mechanism will be preferable to the marketable permit form, or vice versa. It may be politically impossible to reach the marginally optimal level of emissions in a purely win-win-win way, that is, without having to ask environmental users to pay higher taxes in return for some of their gains from reduced emissions. If the pure administration cost of market mechanisms based on emission rights is too high, then some flexible regulatory system may be preferable overall. And if future environmental damage is unknown, property rights to the environment may need to be in the form of long leases rather than freeholds in perpetuity. Yet further complications are likely to arise with imperfect competition, multiple pollutants, and local variations in environmental quality, all of which have been ignored here. Finally, since politics is an art rather than a science, there is not even a guarantee that ensuring that taxpayers, industry and environmental users all get a positive share of the efficiency gains improves the likelihood of market mechanisms being adopted. But this seems to be a useful working hypothesis, worthy of serious consideration by economists and policymakers alike.

Figure 8.1 A win-win-win market mechanism

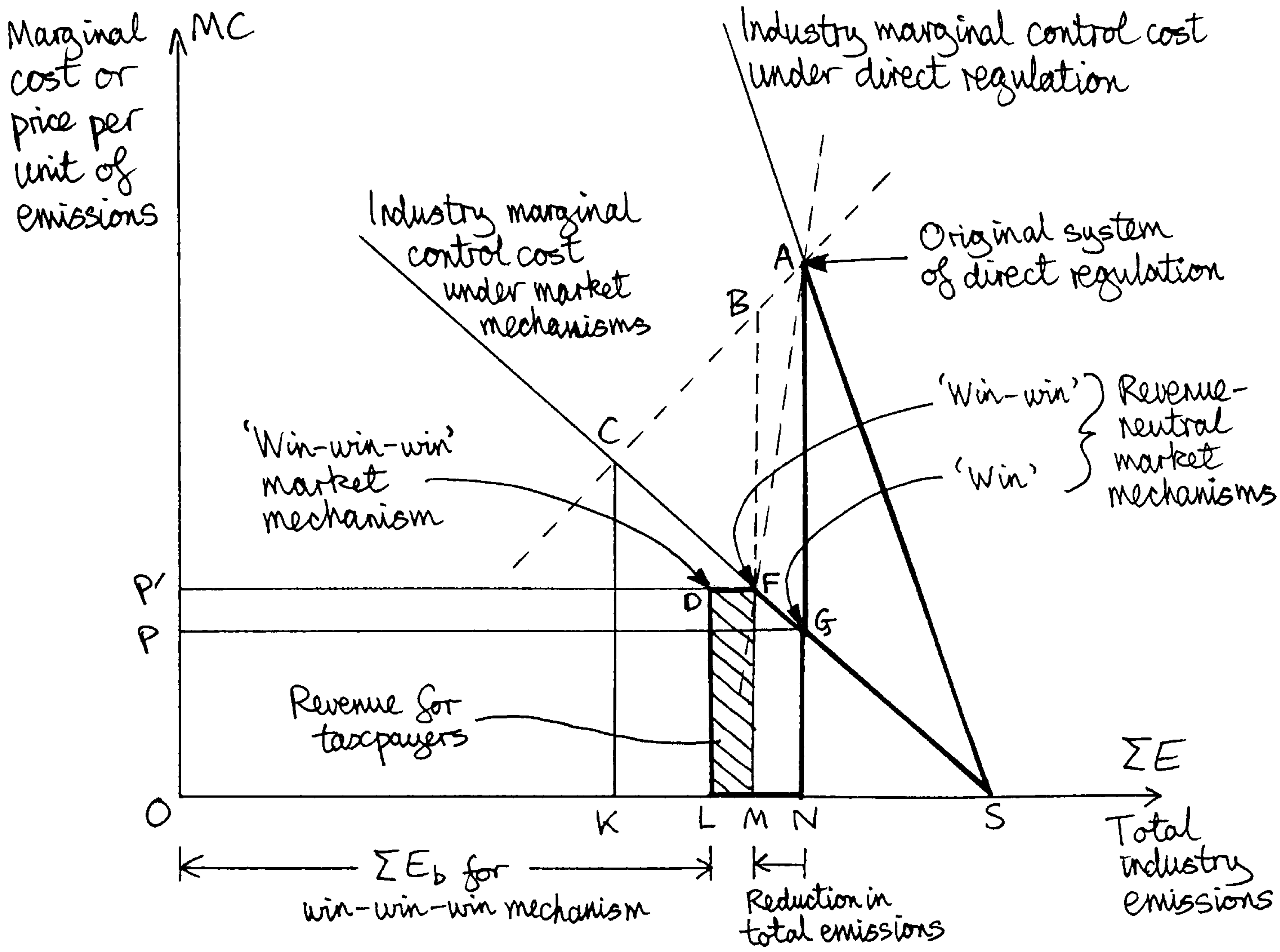


Figure 8.2 Inefficient versus efficient pollution control at the firm level

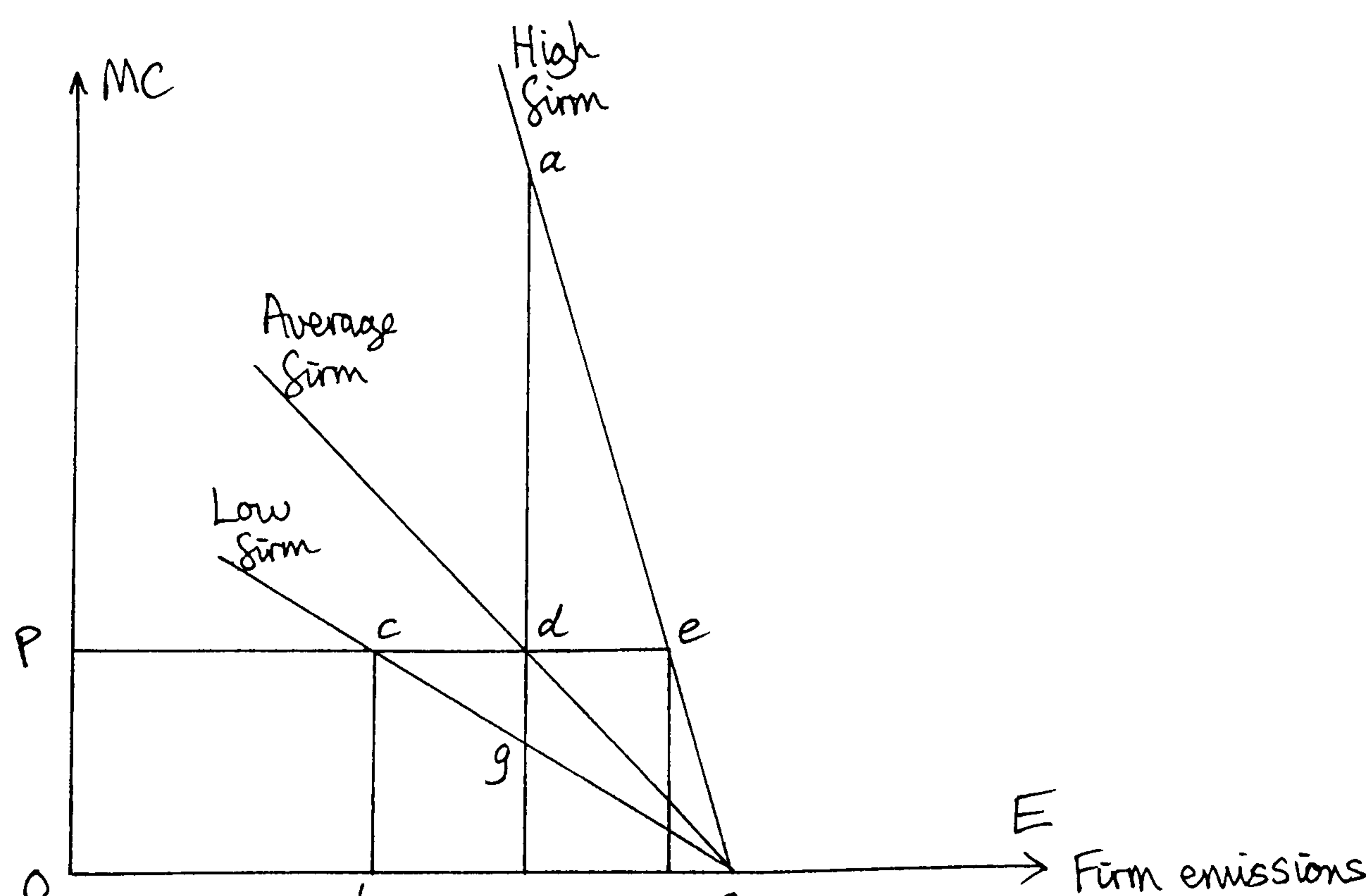


Figure 8.3 A case where the identity of the average firm changes at different emission price levels

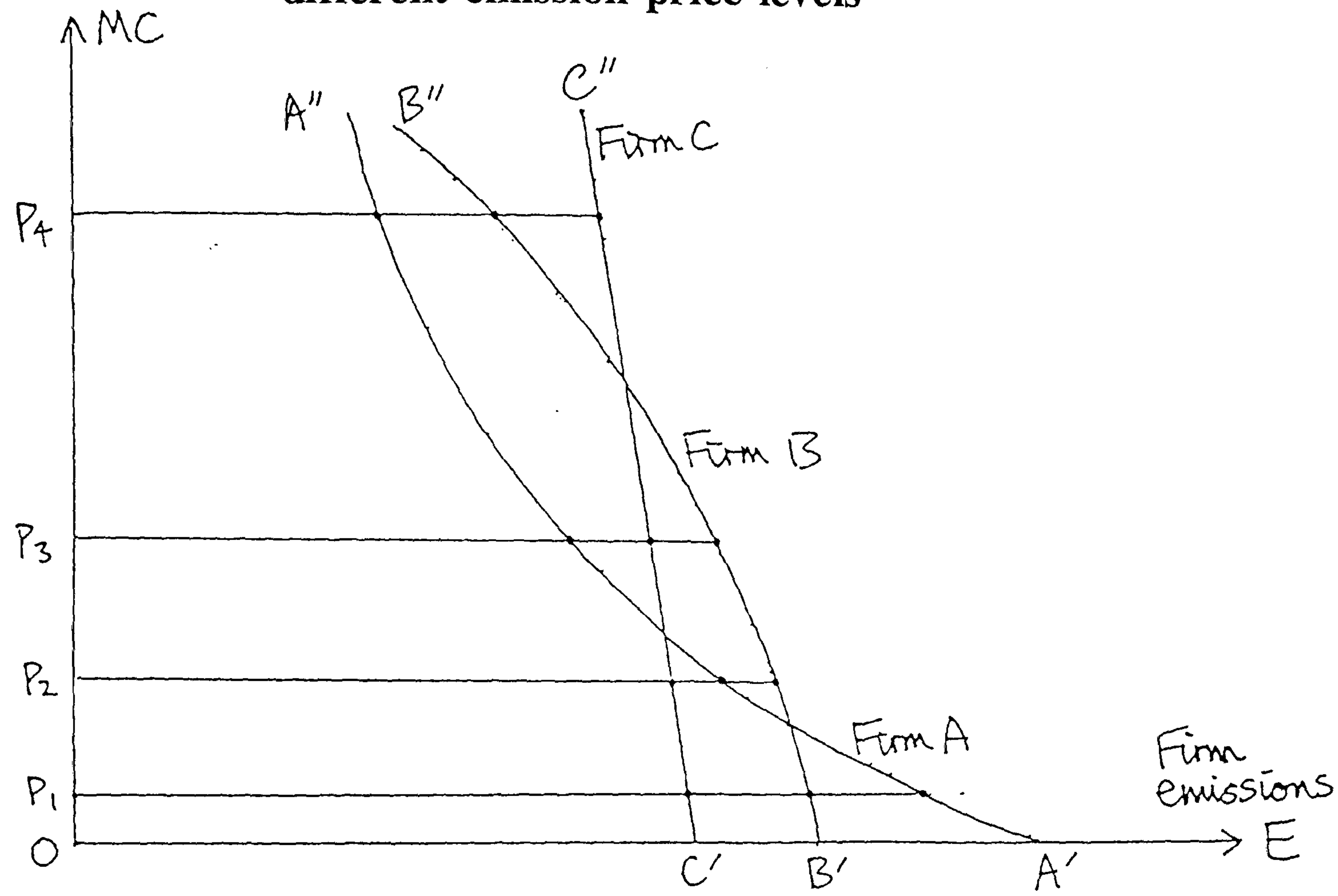


Figure 8.4 A case where a pure market mechanism is of net benefit to an industry

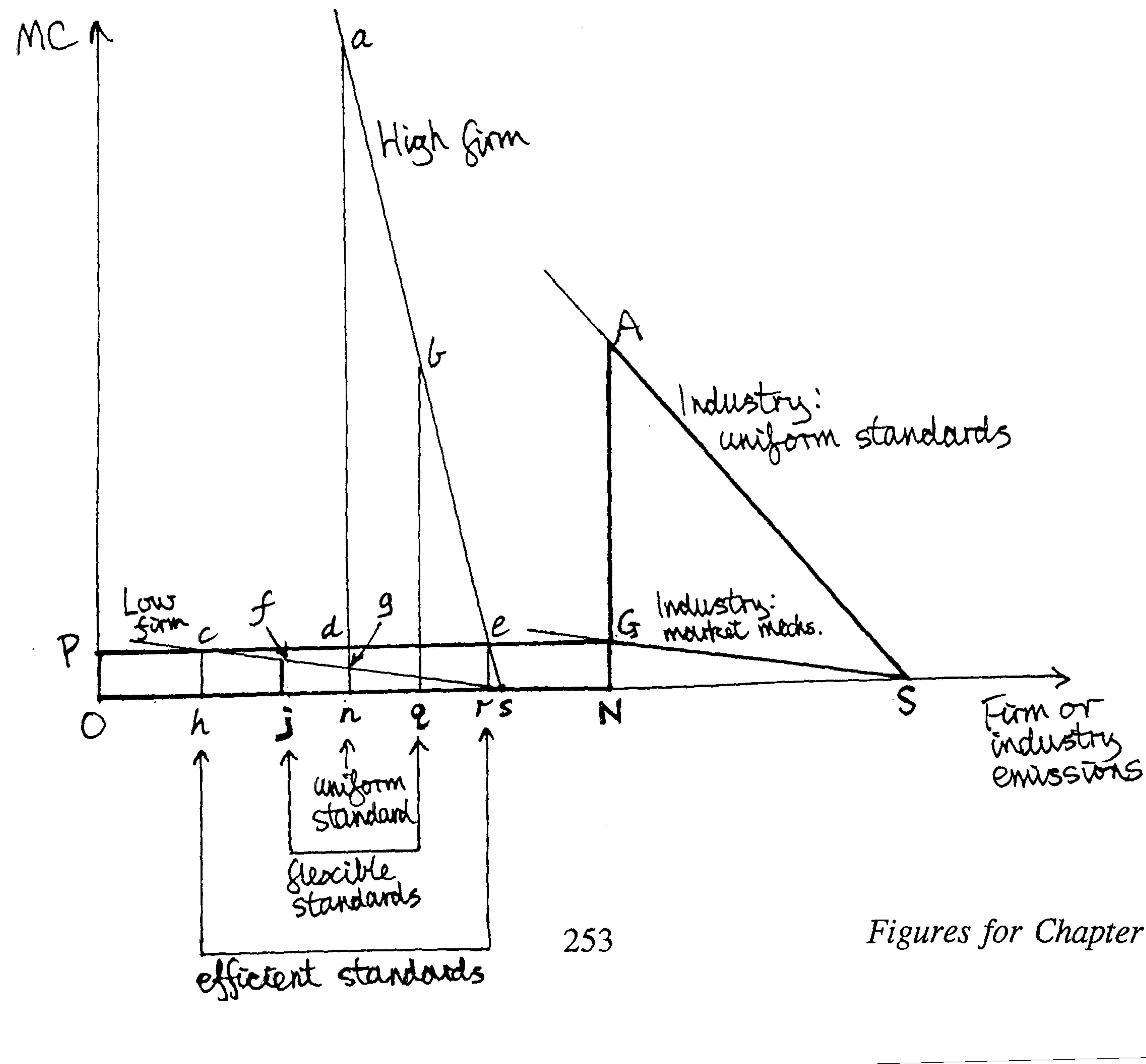
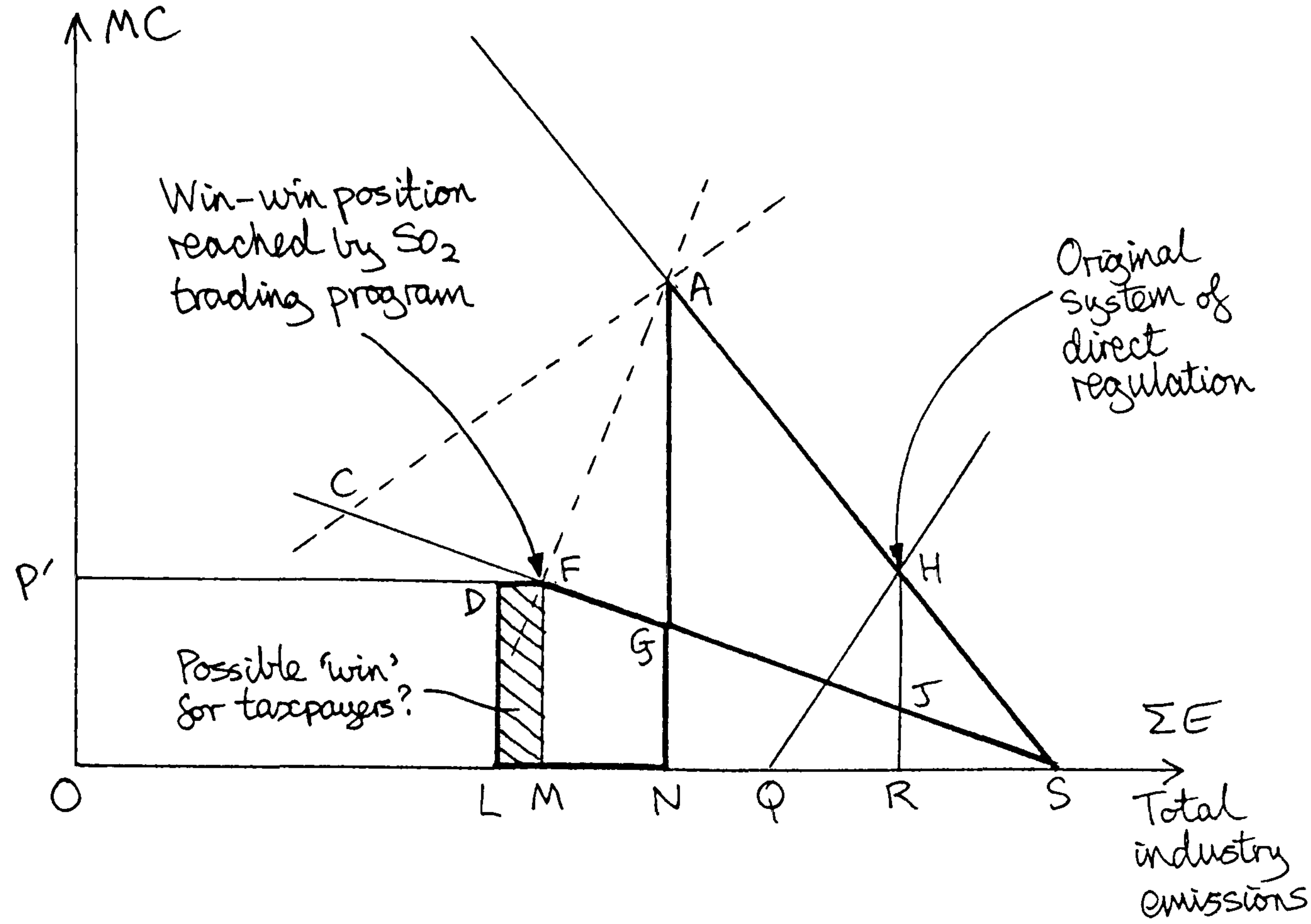


Figure 8.5 Interpretation of the USA's SO_2 trading program as a win-win mechanism



CHAPTER 9

CONCLUSIONS TO PART II

Many theoretical analyses – including Part I of this thesis – automatically assume that it is possible to solve environmental and/or sustainability problems using a per unit tax or charge on the use of environmental resources; and by implication, given well-known symmetry results, that marketable permits, rather than a tax instrument, would also work. By studying the political and practical reality of such market mechanisms, Part II has shown that their applicability is limited by important political and informational factors.

The basic conclusions, which may be unpalatable to some, were that firstly, if the environment is scarce and is to be used optimally, it needs to be owned, like any other scarce resource. The choice between emission control by price (using emission charges) and control by quantity (using marketable emission permits) is then of secondary importance. Chapter 7 showed that the conventional view, that control by price is not symmetric to control by quantity in terms of long run efficiency, is based on an asymmetric (and often unstated) assumption that marketable permits are treated as property rights when a firm enters or exits the industry, whereas subsidies are not. When subsidies are also treated as property rights, a simple symmetry emerges between marketable permits and charges/subsidies. If each instrument embodies exactly the same quantity and quality of emission rights, their short and long run economic effects will be identical, given a number of simplifying assumptions about perfect competition and information.

Secondly, Chapter 8 argued that a move towards economic efficiency via environmental ownership can and should (if it is to be politically attractive) be done in a way which gives 'wins' (net economic benefits) to all the main interest groups who shape environmental policy: environmental users, industry and taxpayers. However, to draw this conclusion alone would be somewhat simplistic. The chapter also showed that firms with near-average emissions will probably lose out from this 'win-win-win' mechanism, and that information costs are a pervasive constraint on policy, and help to explain why inefficient standards formed the original basis of pollution control policy. Information costs mean that a move towards market mechanisms will still face practical problems, even if the policymakers' psychological resistance to the mechanisms is less than it used to be, and even if the political economy of the move is managed in the 'win-win-win' way suggested.

The implications of these findings for further research fall under the two broad headings of political economy and informational economy. Firstly, we need a better understanding of the politics of distributing the efficiency gains from market mechanisms. For any particular case, what are the *de facto* emission rights of existing polluters? How should they be redistributed among taxpayers and environmental users in order to win the most political support for a move to efficient market mechanisms? And why is the 'equal treatment' argument used (if unwittingly) to resist the establishment of emission rights for control by price, but not for control by quantity? (Most policymakers assume that any emission reduction subsidy must, to give 'equal treatment' to firms, be made available to new as well as existing polluters, but do not make the equivalent assumption that marketable emission permits should be given free to new as well as existing polluters.)

Secondly, we need more information about information. For any particular case, how much would it cost to administer the ownership of emission rights? How much extra monitoring and compliance costs will a market mechanism require? How big is the inefficiency of the existing regulatory system in comparison to these information costs? Only if both these two lines of enquiry are pursued will the full potential of market mechanisms of pollution control be appraised realistically, and achieved in practice.

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STANDARD NOTATION AND TERMINOLOGY USED

NOTATION

Like most mathematical economists, I have found the finiteness of the Graeco-Roman alphabet set, and the excess demand for letters such as p (for price, profit, progress or pollution) and t (for time, tax or technology) to be a great source of frustration. The following assignment could no doubt be improved upon, but I hope it will have some mnemonic power, and intended alliterative allusions are noted. (2), (3), etc after the definition means that it is used in Chapter 2, Chapter 3, etc.

$:=$	is defined as
$\propto +$	is proportional to and has the same sign as [used when cancelling out common positive factors during algebraic manipulation]
$\Vert$	end of proof, Q.E.D.
$\dot{}$	denotes first time derivative, d/dt
$\ddot{}$	denotes second time derivative, d^2/dt^2
$\tilde{}$	denotes socially PV-optimal solution which maximises social PV
$\dagger$	denotes sustainable solution in (2), but PV-optimal sustainable (opsustimal) solution in (3)
$\tilde{}\dagger$	denotes socially PV-optimal sustainable solution (2)
$\hat{}$	denotes privately PV-optimal response to policy intervention (2/3/4)
$*$	denotes asymptotic steady state (2)
$\#$	denotes transition (3)
$_0$	denotes at time zero (passim)
$\bar{}$	denotes exogenous parameter (1) // average for whole population (4)
a	age of individual in overlapping generations model (1)
$_a$	denotes asexual (4)
A	exogenous technical productivity factor (2)
b	constant (2)
$_b$	denotes *b*aseline effluent right (7/8)
B	value (*b*enefit) of family sustainability (4)
c	per capita *c*onsumption (2)

C total *C*onsumption (2/3) // cost function (7)

$_C$ denotes $\partial/\partial C$, or connected with *C*onsumption

D utility *D*iscount factor (in 2-period model) (4)

$_{es}$ denotes (individually) *e*xternally *s*ustainable (4)

E *E*ffective stock of resource (1) // *E*ffluent emitted by a firm (7/8)

f resource-intensive form of production *f*unction (2/3) // particular family (4)

F production *F*unction (2/3)

g growth function of resource stock (1) // propensity to consume out of cake resource ('*g*reed') (4)

$_G$ denotes historically given initial value (3)

h constant (2)

H *H*amiltonian (3)

$_H$ denotes connected with *H*artwick's Rule (3)

$_i$ denotes *i*ndividually sustainable (2, Appendices)

$_{is}$ denotes (individually) *i*nternally *s*ustainable (4)

J sustainability *J*ump function (4)

K total capital (2/3)

$_K$ denotes $\partial/\partial K$, or connected with capital (2/3)

L total quantity of *L*abour (2/3)

m ratio of marginal cost between two firms (8)

^m_t denotes *m*aximin solution available at time t (2/3)

M marginal control cost at industry level (8)

N number of periods that an individual lives (1)

p *p*rices of all capital and resource stocks in terms of consumption (3)

$_P$ denotes connected with the *P*eak time of a PV-optimal path (3)

PV *P*resent *V*alue of utility (2/3/4)

PV_t current PV = PV of a utility path from time t onwards (2)

PVC *P*resent *V*alue of *C*onsumption (3)

Q total *Q*uantity of output (of consumption good) (2)

r *r*ate of interest (2/3)

R rate of *R*esource extraction (2/3)

$_R$ denotes $\partial/\partial R$, or connected with *R*esource stock and flow (2)

$_{ss}$ denotes *s*ocially *s*ustainable (4)

s	per capita *s*tock of resource (2) // time variable in integral (3)
S	total *S*tock of exhaustible resource (2)
$_S$	denotes $\partial/\partial S$, or connected with resource *S*tock (2)
t	*t*ime (passim)
T	upper *T*ime limit of integration, transition *T*ime (3)
U	net benefit to society at a particular time (1) // instantaneous *U*tility of representative, infinitely-lived agent (2/3/4)
V	lifetime utility of an individual (1) // aggregate consumption wealth (3) // effluent charge rate (7/8)
W	intergenerational social welfare function (ISWF) (1) // overall intertemporal *W*elfare of a parent (4)
x	time subscript (1) // K/R (2/3)
$_x$	denotes connected with x (2/3)
X	bequest which child's mate brings (unknown to parents) (4)
y	propensity to consume of child's family (unknown to parents) (4)
Y	net national product $= C+\boldsymbol{p}.\dot{\Sigma}$ (3)
z	small number (2)
Z	net national welfare $= U+\boldsymbol{\pi}.\dot{\Sigma}$ (3)
α	elasticity of capital (but not resources) in production (2/3)
β	Q/K (2)
γ	exogenous parameter (1) // R/S (2)
Γ	exogenous technical or biological *g*rowth factor (4)
δ	utility *d*iscount rate (2/3)
ϵ	*e*nvironmental preference parameter (2)
ζ	$-\dot{S}/S$ in Sec 3.3 (2)
η	*e*lasticity of marginal utility of consumption (2/3)
θ	resource depletion rate in socially PV-optimal cake-eating model (2)
Θ	algebraic expression (3)
κ	$\tau\nu/(\nu+\epsilon)$ (2, Appendix)
λ	Lagrange multiplier (2) // general utility discount factor (3)
μ	instantaneous utility of finitely-lived agent in overlapping generations framework (1) // small number (3, Appendix)
$_\mu$	denotes free *m*arket solution path (2/4)

ν material values parameter (2)

ξ $\dot{K}/K$ in Sec 3.4 (2)

π current value *p*rice of capital or resource stock in terms of utility (2/3)

$\boldsymbol{\pi}$ current value *p*rices of all capital and resource stocks in terms of utility (3)

Π *p*roduction possibility set (3)

ρ asymptotic f' (3)

$^{\rho}$ denotes pe*r*turbed path (2, Appendix) // denotes *r*eference level (4)

σ elasticity of *s*ubstitution between capital and resources (2/3)

Σ vector of all capital and resource *s*tocks (3)

$_{\Sigma}$ denotes resource conservation incentive in steady state case (2)

τ rate of exogenous, exponential *t*echnical progress (2)

ϕ tax (*f*ee) rate on particular variable (2/3/4)

Φ strength of tax policy (2)

Φ_C $-\dot{\phi}_C/(1+\phi_C)$ (2)

Φ_R $\hat{f}'\phi_R-(\dot{\phi}_R+\phi_S)/A$ (2)

Φ_x $\Phi_R-\phi_K\phi_R/A$ (2)

ψ arbitrary time exponent (2) // price of consumption in utility terms (3)

Ψ aggregate wealth $= Z(0)/\delta + \int_0^t \boldsymbol{\pi}.\dot{\Sigma}$ (3)

ω exponent of labour (2) // small number (3, Appendices)

Ω lump sum refunds (2) // gross wealth (3)

TERMINOLOGY

aggregate investment (3)	$\pi.\dot{\Sigma}$: the total value of net investment in all capital stocks and depletion of all resource stocks
aggregate wealth (3)	integral of aggregate investment over time
current maximin (3)	maximum constant level (of consumption or utility) that is attainable, given current assets, from the current time onwards
current PV (3)	PV of utility path from current time t onwards
Dasgupta-Heal economy (3)	a Weitzman economy whose only productive inputs are one type of capital and one type of non-renewable resource
DH74 (3)	Dasgupta and Heal (1974), "The Optimal Depletion of Exhaustible Resources"
DH79 (3)	Dasgupta and Heal (1979), *Economic Theory and Exhaustible Resources*
free market path (2/4)	path that a private PV-maximising individual follows, given the absence of any policy intervention
individual sustainedness (2/4)	non-declining utility achieved by one agent, when all others may have declining utility
ISWF (2)	intergenerational social welfare function
net national product (3)	consumption, plus aggregate investment measured at consumption prices
NNP (3)	net national product
net national welfare (3)	utility plus aggregate investment measured at utility prices
NDU (1/2/3/4)	non-declining instantaneous utility of a representative person
policy path (2/3/4)	path that a private PV-maximising individual follows, given the existence of policy intervention
PV (1/2/3/4)	present value of utility
PVC (3)	present value of consumption

SD (1/2/3/4)	sustainable development = NDU forever
SSCI (1)	Social Science Citations Index
sustainability (1/2/3/4)	the ability to achieve NDU forever
sustainedness (1/2/3/4)	NDU forever
unsustainability (2/3/4)	the inability to achieve NDU forever
Weitzman economy (3)	an economy with a smooth and convex production possibilities set, utility dependent only on (scalar) consumption, and *no technical progress*

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